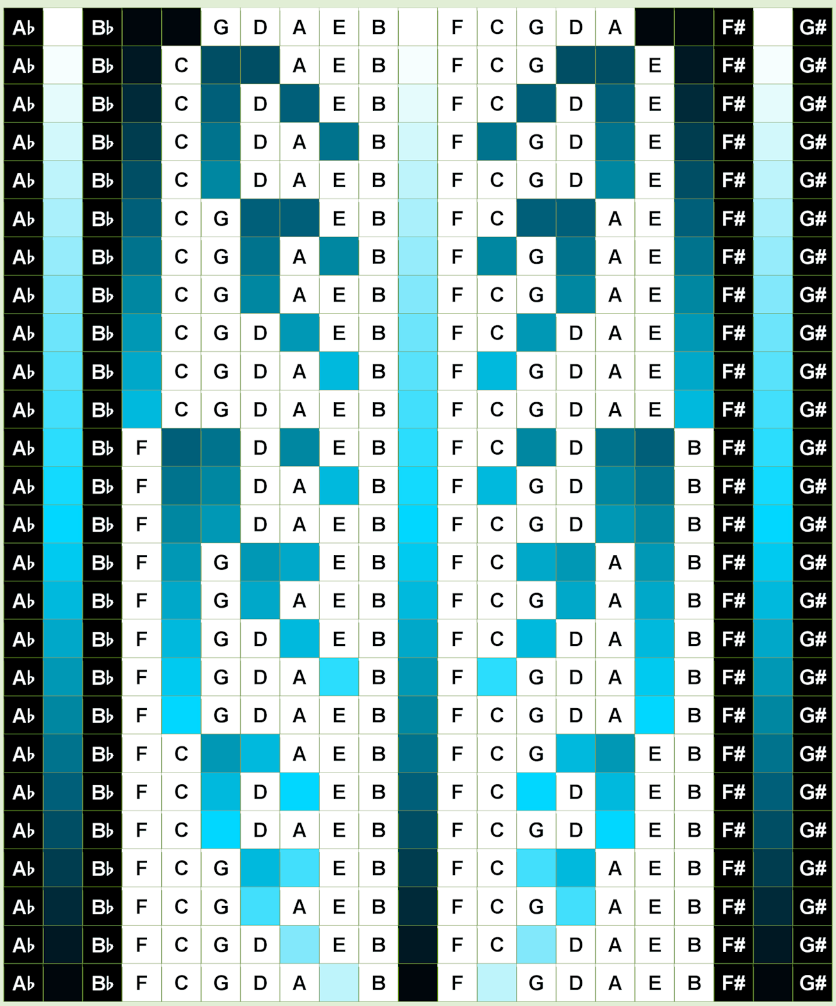
Harmonic Processions

The Hierarchy of Chords and Scales: A Theory of Consonance and Dissonance



Dosia McKay

Harmonic Processions

The Hierarchy of Chords and Scales:
A Theory of Consonance and Dissonance

Dosia McKay

Gavia Music

About the PDF Edition

This free PDF edition is provided as a courtesy to introduce the core theoretical framework of the Harmonic Processions theory and is intended for personal use only. The complete system—including all 40 chapters and all 48 reference tables—is available exclusively in the printed edition, which can be purchased at www.DosiaMcKay.com.

ISBN 978-1-7341225-5-8

Library of Congress Control Number: 2026908898

Cover, interior design, and illustrations by the author
Published by Gavia Music, Knoxville, Tennessee, 2026

Chapters

Harmonic Processions Tables

The Natural Harmonic Processions

The Modal Harmonic Processions

The Dissonance-Gradient Harmonic Processions

Other Tables

1. The Harmonic Processions Theory

The Harmonic Processions Theory is a system for classifying all 350[1] possible sets—commonly referred to as chords and scales—found in Western[2] music. The name reflects the system's structure, in which harmonies process sequentially from simple to complex, revealing shared origins and interrelationships. The processions traverse a gradient from consonance to dissonance, organizing sonorities by distinct harmonic flavors, or modalities.

Guided by mathematical principles, the theory of Harmonic Processions unveils an expressive world that is elegant in its logic. It serves as a powerful tool for composers, music theorists, musicologists, and music-theory enthusiasts seeking to explore the nature of musical harmony. More than a theoretical framework, it is also a practical reference—a catalog of harmonic swatches, a harmonic color wheel—designed to illuminate the character and potential of all chords and scales.

As we enter the world of harmonic structure, we cannot help but marvel at the sequences and patterns we encounter. The twelve tones of the chromatic scale—like the twelve months of the year, the twelve hours on the clock face, the twelve signs of the Zodiac, the Twelve Apostles, or the twelve hues on the color wheel—point to a mystical visual language echoed in snowflakes under a microscope, in fractals, and in sacred geometry. In this world, we are not the creators; we are merely humble observers.

2. Why This Book?

Painters have color swatches, color wheels, and gray scales. Architects consult reference books on the properties of steel, concrete, and timber. Chemists have the Mendeleev's periodic table of elements. But what do composers have to guide their harmonic explorations? How do they find that perfect chord or scale—one suitable for the occasion, not too cliché but also not too jarring—something with a base of vanilla, a touch of melancholy, yet still bright and open?

Music students learn about harmony gradually from various sources. As children, they encounter happy and sad chords (major and minor). Guitarists quickly discover the richness of added 7ths and 9ths, while the pianists devote hours to practicing the harmonic and the melodic minor scales. In college, they meet the pentatonic scale and the diatonic modes. Venturing further—perhaps while playing in a wind ensemble or orchestra—they encounter the octatonic and the whole-tone scales, unlocking yet another layer of harmonic depth.

Many college programs require music majors to take post-tonal theory, a course that dazzles and confounds with interval-class vectors, hexachordal combinatoriality, and transpositional symmetries. Amid this strange and exhilarating landscape, one steadfast element emerges: the list of set classes—the complete catalog of 350 possible chords and scales encountered in Western music. Among them are the familiar major chord and major scale, the Prometheus chord, the octatonic, the chromatic scale, and many others, repeatedly used by

[1] Not counting the unison or the empty set.
[2] The theory of Western music is based on the equal division of the octave into twelve pitches (well-tempered tuning).

composers and dissected by theorists. Still many more remain like distant stars: identified by a number but unnamed and unexplored. With barely a six-digit number (interval-class vector) assigned to each set class, there is little to indicate the character, the modality, the flavor, or the level of dissonance of these sonorities. If the list of set classes is meant to be the composer's periodic table of elements or a color swatch book, it is not particularly inviting.

Where do composers turn when they want to convey sorrow, bitterness, bliss, curiosity, joy, or ambiguity? Most rely on a mixture of instinct and luck, experimenting with harmonies at the piano until something interesting emerges. Many look to other genres for inspiration. They juxtapose tried-and-true harmonic elements in hopes of finding a new angle. Adopting the favorite chords and scales of other composers has long been a standard practice. Yet the fresh sound—the ultimate harmonic progression—often feels elusive, always seeming just out of reach, promising to reveal itself in the next composition. How, then, can composers claim those unnamed distant stars for themselves?

The purpose of this book is to explore every chord and scale and to catalog them in a way that makes their content, qualities, and relationships easy to reference, intuitive to understand, and practical to apply in creative work. If you have ever wished for a color wheel of harmony—a true reference guide to sonic possibilities—this is where your search ends and creativity begins.

3. How to Use This Book

This book assumes that the reader is already familiar with basic music-theory concepts such as scales, chords, intervals, accidentals (sharps and flats), and transposition. The ability to read music notation in both treble and bass clefs is also beneficial. Knowledge of post-tonal theory is helpful, although not essential. As more advanced ideas are introduced, they are explained in an accessible manner.

The heart of the book lies in the enclosed tables, which invite the reader to bypass the introductory material and immediately explore new chords and scales through improvisation or composition. Readers who prefer a deeper understanding of the methodology will find detailed explanations, examples, and exercises in the preceding chapters.

4. Is It a Chord or a Scale?

The Prometheus Chord and Scale

A chord is a collection of notes stacked vertically and sounded simultaneously, whereas a scale is a collection of notes arranged in ascending or descending order, with the first note serving as its tonal center.

Let us examine the Prometheus chord (also known as the Mystic chord) and the Prometheus scale, 106♭[3] (F 6-34A[4]), a favorite of the Russian composer Alexander Scriabin during his later creative period in the early twentieth century. The name of this chord (or scale) was coined by the musicologist Leonid Sabaneyev after Scriabin used it extensively in *Prometheus: The Poem of Fire*, although Scriabin himself referred to it as the chord of the *pleroma*—the totality of divine powers.

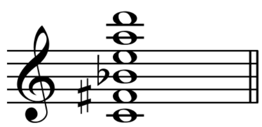


Figure 4-1. The Prometheus chord 106♭ (F 6-34A)

Figure 4-2. The Prometheus scale 106♭ (F 6-34A)

In the examples above (Figure 4-1 and Figure 4-2), the Prometheus chord and the Prometheus scale contain the same notes. Both assume C as the tonal center, since each begins on C. These are traditional voicings of the chord and the scale, but either one can be presented in different configurations. For example, the voicing of the chord may be altered, as illustrated in Figure 4-3.

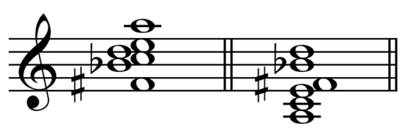


Figure 4-3. Alternative voicings of the Prometheus chord

Likewise, we can rotate the notes of the scale so that it beings on E or A (Figure 4-4 and Figure 4-5).

Figure 4-4. The Prometheus scale beginning on E

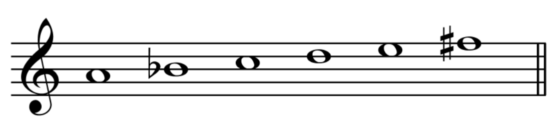


Figure 4-5. The Prometheus scale beginning on A

[3] Harmonic-Processions numbers are discussed in Chapter 25.
[4] Forte numbers are discussed in Chapter 39.

When considering the preceding configurations, which model proves more effective—the chord or the scale? And which voicing is most compelling: the one beginning on C, E, F#, or A? We will consider these questions but first let us examine other sonorities.

Long and Short Scales vs Chords

Besides the Prometheus scale, there are many other six-note and seven-note scales, such as the whole-tone scale, 197S (F 6-35), the harmonic minor scale, 146♭ (F 7-32A), and the major scale, 42S (F 7-35), familiar to anyone who has studied a musical instrument. These six- and seven-note scales naturally lend themselves to presentation as a succession of ascending or descending notes. This is how they are practiced, after all (Figure 4-6 and Figure 4-7).

Figure 4-6. Whole-tone scale on C, 197S (F 6-35)

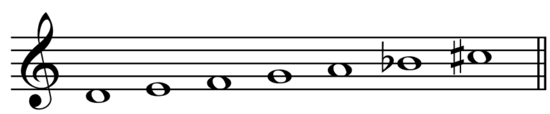


Figure 4-7. D-harmonic minor scale 146♭ (F 7-32A)

We may be accustomed to seeing these scales spelled out in this manner, but nothing prevents us from arranging them into chordal structures, even though these are more cumbersome to analyze (Figure 4-8).

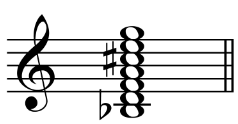


Figure 4-8. A possible voicing of the D-harmonic minor scale when presented as a chord

On the other hand, three-part and four-part harmonies—standard in much of popular and sacred music—naturally lend themselves to quick assessment as chords, without being rearranged into an ascending or descending succession of notes. But couldn't these triads and tetrachords also be classified as scales—very short ones? (Figure 4-9 and Figure 4-10).

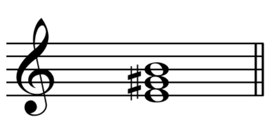


Figure 4-9. E-major triad 8# (F 3-11B)

Figure 4-10. A 3-note scale based on the E-major triad 8# (F 3-11B)

Is it better to present note collections as chords or as scales? Does the choice depend on the number of notes, and if so, what is the optimal number for each? Perhaps musical context should guide our approach: a melodic context suggests a scale, while a harmonic context favors chords. What, then, constitutes best practice? These are valid questions and many aspiring music theorists enjoy debating them. However, in the context of Harmonic Processions, distinguishing between a chord and a scale is of secondary importance. A more useful approach is to treat both simply as sets of notes—or, more succinctly, as sets.

Set

The word *set*—likely borrowed from mathematical set theory—has been widely adopted by theorists of post-tonal music because it denotes a collection of distinct elements. In the context of Harmonic Processions, we will use terms such as *scale*, *chord*, *sonority*, *harmonic area*, and *collection* interchangeably, with the understanding that all of these refer to a set of notes—or, more simply, a *set*.

The advantage of using the word *set* over *chord* or *scale* lies in its egalitarian treatment of the notes it contains; it does not presuppose a tonal center. For example, the set C–D–E–F–G–A–B, 42S (F 7-35), need not default to the C-major scale. It may function as E Phrygian (Figure 4-11), A Aeolian (Figure 4-12), or any modal configuration the composer chooses.

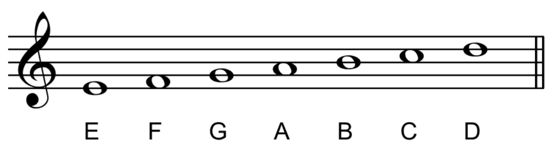


Figure 4-11. E-Phrygian mode

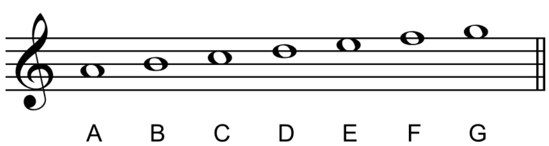


Figure 4-12. A-Aeolian mode

Likewise, the set C–E–G–A, 11S (F 4-26), does not have to function as a C-major chord with an added sixth, nor is the composer obligated to assign C as its root. It does not have to be a chord at all; it can just as easily be used to shape a melodic line. Once again, the word *set* removes theoretical constraints and allows the composer to arrange its notes freely (Figure 4-13 and Figure 4-14).

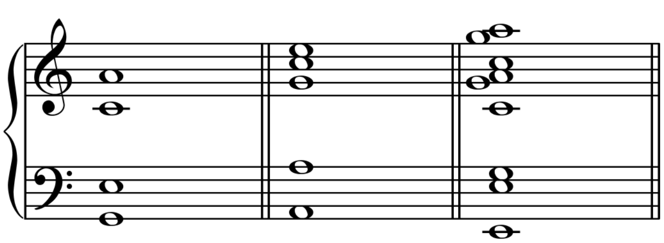


Figure 4-13. Chords constructed with set C–E–G–A

Figure 4-14. A melodic line composed with set C–E–G–A

Nevertheless, since the words *scale* and *chord* are so often used colloquially, we will not discard them. After all, it is easier to say *hexachord* than *six-note set*. However, it will be understood that our primary interest lies in the contents of the sets rather than in their theoretical presentation or interpretation.

And since the terms *triad*, *tetrachord*, and *hexachord* have now been introduced, let us establish the following vocabulary:

Number of notes in a set	Name of the chord	Name of the scale
2	Interval or Dyad	Ditonic
3	Trichord or Triad	Tritonic
4	Tetrachord	Tetratonic
5	Pentachord	Pentatonic
6	Hexachord	Hexatonic
7	Heptachord	Heptatonic
8	Octachord	Octatonic
9	Nonachord	Nonatonic
10	Decachord	Decatonic
11	Undecachord	Undecatonic
12	Dodecachord	Chromatic

In Practice

- Using an instrument (or without one), choose any five notes that form an interesting set. Compose a twelve-measure melodic line in 4/4 meter using only those five notes in any octave. Alternatively, improvise a melodic line on an instrument, limited to the same five notes in any octave.

- Choose six distinct four-note sets that form interesting chords. Notate them in the treble clef or in keyboard notation (treble and bass clefs together). Experiment with different voicings, inversions, and registers. Don't worry about their functions or theoretical labels, but if any of them strike you as particularly compelling, feel free to give them nicknames.

- Choose three different scales you already know. Arrange each one vertically as a chordal structure and notate it in keyboard notation, exploring various voicings and spacings. Ensure that every note of the scale is represented in the chord.

5. Note and Pitch

The words *note* and *pitch* are often used interchangeably, but the distinction between them is important. *Pitch* refers to a specific frequency in a specific register (octave)—for example, F2 or G#5. *Note*, by contrast, refers to all pitches that share the same letter name, regardless of octave. In other words, when we discuss the note B♭, we include all B♭ pitches in any register and on any instrument (Figure 5-1). And while post-tonal theory prefers the term *pitch class*, we will mostly use the word *note*.

Figure 5-1. Note B♭ in various registers

The theory of Harmonic Processions does not distinguish between registers, the positions of notes within chords (voicing), or the spacing between them. Our primary focus is the content of sets, their characteristics, and the relationships among various sets.

It is obvious that a D-major chord in open voicing in the upper register of the piano sounds bright and clear, while the same chord in closed voicing in the low register is perceived as muddy (Figure 5-2). Nevertheless, it still retains the essential qualities of a major chord. It is this essence that concerns us most in our explorations.

Figure 5-2. D-major chord in open spacing and closed spacing, in different registers

In Practice

- Listen to a G-major chord played on the piano, guitar, or another instrument, exploring as many voicings and registers as possible. Does it sound different each time? Which qualities remain consistent? Does it project a characteristic feeling or flavor? Does it convey a particular emotion or sentiment? Does it express something unique to itself? Compare your impressions with those of the G-major 7th chord, the G-minor chord, and the G-minor 7th chord. Catalog your observations in a table.

6. Enharmonic Equivalence

In Western music, the equal division of the octave into twelve notes, combined with the many available transposition options, requires that all notes be interpreted harmonically according to their context. For example, the seventh note (the leading tone) of the A-major scale is G# (Figure 6-1), while the third note of the F-natural minor scale is A♭ (Figure 6-2), even though both are played on the same key of the piano. These notes are enharmonically equivalent.

Figure 6-1. A-major scale with G# as the leading tone

Figure 6-2. F-natural minor scale with A♭ as the third scale degree

Likewise, in the F#-major triad the third is spelled A#, while in the G minor triad, the third is spelled as B♭ even though, enharmonically speaking, they represent the same note (Figure 6-3).

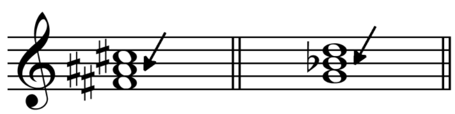


Figure 6-3. A# in F#-major triad vs B♭ in G-minor triad

We recognize that in various contexts these notes may be performed differently, with slight adjustments in tuning. Any musician who has collaborated in an ensemble—whether playing a string, woodwind, or brass instrument, or singing—knows that the twelve-tone system is not fixed with absolute rigidity and that minute intonational adjustments are often necessary. However, within the theory of Harmonic Processions, we will subscribe to the principle of enharmonic equivalence and will assume that G# is equivalent to A♭, and A# to B♭, with respect to their positions among the twelve divisions of the octave.

Many examples in this book list notes using the enharmonic spelling that best reflects the set to which they belong, but it is understood that the composer or theorist is free to reinterpret these notes according to their preferred harmonic context. Consequently, some enharmonic respelling may be required.

In Practice

- Correct the enharmonic spelling of the notes within the following sets:
  - E-major scale: E–F#–A♭–A–B–D♭–D#–E
  - D-major 7th chord: D–G♭–A–D♭
  - A♭-major chord: G#–B#–D#
  - Whole-tone scale: E–G♭–A♭–A#–C–D

7. Interval and Interval Class

Interval

An interval, in the simplest terms, is the distance between two pitches. In theory, there could be an infinite number of intervals if we imagine an infinite continuum of possible frequencies. However, within the theory of Harmonic Processions, our concern is not the precise acoustic distance between two pitches but the harmonic relationship between notes. For example, in the case of the set C–G, we are interested in the relationship between these notes regardless of register, voicing, or inversion (whether C lies above G or G above C). To quantify such relationships, we will rely on interval classes.

Interval Class

The term *interval class*, commonly used in post-tonal music theory, conveys the idea that when discussing harmonic relationships between notes, it is most efficient to describe intervals in terms of their smallest possible form.

For example, the interval between C3 and G4 is a large interval of perfect twelfth (P12) (Figure 7-1). If we transpose G4 down an octave to G3, the interval becomes the more manageable perfect fifth (P5). We can simplify it further by inverting the pitches, which yields a perfect fourth (P4). The perfect fourth is the smallest interval that can exist between C and G; no further transposition or inversion can reduce the distance between these two notes.

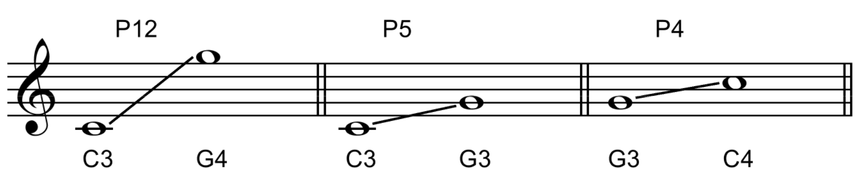


Figure 7-1. The interval class of perfect fourth (P4)

Thus, the perfect fourth is the interval chosen to represent a class—a group—of intervals of various sizes that can all be reduced to the perfect fourth. Practically, this means that whether we discuss the perfect twelfth (P12), the perfect fifth (P5), or the perfect fourth (P4), all are understood as belonging to the interval class of the perfect fourth.

There are only six interval classes, and they can be deduced by mapping all intervals within the octave and eliminating those that can be further reduced through inversion.

Below is a list of these intervals, arranged in pairs that show each smaller interval alongside its larger inversion (Figure 7-2). Notice that the tritone (TT) inverts to another tritone.

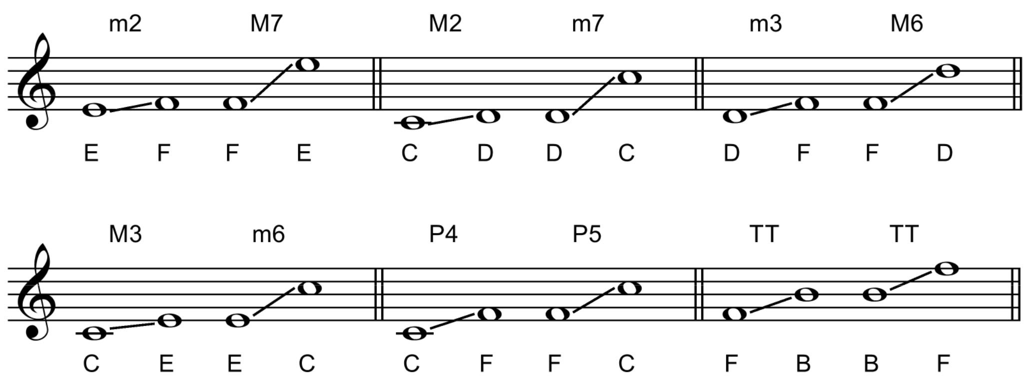


Figure 7-2. Intervals and their inversions in music notation

The table in Figure 7-3 references the same concept.

Small Interval	**Large Interval**
m2 Minor second	M7 Major seventh
M2 Major second	m7 Minor seventh
m3 Minor third	M6 Major sixth
M3 Major third	m6 Minor sixth
P4 Perfect fourth	P5 Perfect fifth
TT Tritone	TT Tritone

Figure 7-3. Intervals and their inversions table

The left column of the preceding table illustrates the six interval classes, which can be used to express all harmonic relationships between notes. In other words, when we encounter a major seventh (M7), we will treat it as a minor second (m2), and when we see a minor sixth (m6), we will reduce it to a major third (M3), and so forth. As before, notice that the tritone (TT) is a special case: its inversion is another tritone.

The six interval classes, in order of their size, are as follows (Figure 7-4):

Interval Class	m2	M2	m3	M3	P4	TT
Semitone Count	1	2	3	4	5	6

Figure 7-4. Interval classes

In the context of post-tonal theory, interval classes are always expressed as semitone counts from 1 to 6. In our exposition, however, counting semitones would introduce an unnecessary layer. For our purposes, it is more intuitive and efficient to express interval classes using traditional interval names rather than semitone numbers.

While augmented and diminished intervals are useful for describing harmonic motion and tendencies toward tonal centers, we will refrain from using them here for the sake of clarity. Thus, an augmented second (2<) will always be treated as a minor third (m3), and a diminished fifth (5>) will be treated as the tritone (TT), regardless of enharmonic spelling. Once again, our interest lies in the content of a note set, not in its enharmonic interpretation.

In Practice

- In music notation, write the following intervals and determine their interval class. Remember to invert them, if necessary, to find the smallest interval:
  - G–C#
  - C#–C
  - E–F
  - A–F#
  - B♭–F

8. The Overtone Series

Oscillation

The overtone series (also known as the harmonic series) refers to the set of frequencies that naturally occur when a pitch is produced by an acoustic instrument or the human voice. Whether initiated by an oscillating string or a column of air, this physical phenomenon generates a fundamental frequency accompanied by a series of sympathetic overtones—also called harmonics or partials—which occur at integer multiples of the fundamental.

In other words, the vibrating medium (string or air column) oscillates not only along its full length but also simultaneously in fractional segments—at 1/2, 1/3, 1/4, and so on—producing higher frequencies (Figure 8-1). These oscillations form standing waves that interact with the surrounding air to create audible sound waves. The resulting waveforms may reinforce or interfere with one another, shaping the timbre of the sound.

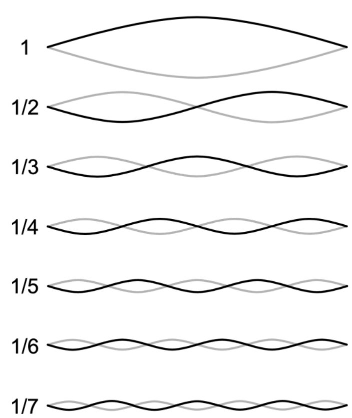


Figure 8-1. Oscillation segments of a vibrating string

Every pitch, in theory, is accompanied by an infinite number of overtones. However, the higher the overtone (that is, the shorter its wavelength), the weaker its amplitude. Many of the upper partials become imperceptible to the human ear because of their diminishing intensity. Below is a diagram of the fundamental pitch C1 (labeled 1) and its harmonic series up to the 16th harmonic (Figure 8-2).

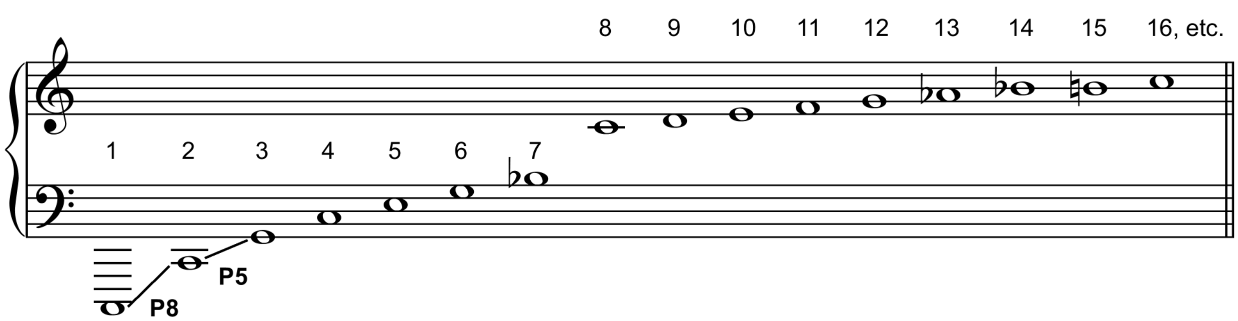


Figure 8-2. Overtone series of the pitch C1

Notice that the interval of the perfect fifth (P5) appears immediately after the octave (P8), making it the second most prominent, most sympathetic, and most easily perceived harmonic in the overtone series.

Interval Ratios

So far we have discussed the harmonic content of a single pitch (C1), but what happens when two or more pitches are sounded simultaneously? Their fundamentals and all associated harmonics interact in a complex web of constructive and destructive interference, governed by the frequency ratios among all participating tones. This interplay can produce a range of sonic outcomes—some perceived as consonant (pleasing and stable), others as dissonant (tense or unstable).

Generally, the simpler the ratios between the frequencies of the fundamentals and their overtones—such as 2:1, 3:2, or 4:3—the more consonant the resulting interval. More complex or non-integer ratios tend to produce dissonance due to less harmonic alignment and more phase instability.

For example, the interval of an octave corresponds to a 2:1 frequency ratio: if the lower note oscillates at 200 Hz, the upper note will oscillate at 400 Hz. This ratio causes significant overlap in the overtone series, leading to constructive interference and a reduction in dissonance. The auditory system interprets this alignment as a pleasing and stable sound, which is why octaves are universally recognized as consonant intervals.

Take a look at the frequencies associated with each pitch and their overlap:

- Lower pitch: 200 Hz, 400 Hz, 600 Hz, 800 Hz, 1000 Hz, 1,200 Hz, 1,400 Hz, 1,600 Hz, 1,800 Hz, etc.
- Upper pitch: 400 Hz, 800 Hz, 1,200 Hz, 1,600 Hz, 2,000 Hz, 2,400 Hz, 2,800 Hz, 3,200 Hz, 3,600 Hz, etc.

Now consult the table in Figure 8-3 to examine the frequency ratios of all intervals within the octave, as tuned in just intonation—a system that aligns with the harmonic series. Once again, after the octave (2:1 ratio), the perfect fifth (3:2 ratio) presents the next simplest ratio and is therefore most consonant.

In contrast, the tritone, with its ratio of 45:32, produces fewer overlapping harmonics and is perceived by the auditory system as less stable and more tense.

Whether in the overtone series or in intervallic ratios, the perfect fifth (P5) consistently emerges—after the octave—as the most consonant and acoustically sympathetic interval relative to the fundamental pitch.

Abbreviation	Interval	Ratio
U	Unison	1:1
m2	Minor Second	16:15
M2	Major Second	9:8
m3	Minor Third	6:5
M3	Major Third	5:4
P4	Perfect Fourth	4:3
TT	Tritone	45:32
P5	Perfect Fifth	3:2
m6	Minor Sixth	8:5
M6	Major Sixth	5:3
m7	Minor Seventh	9:5
M7	Major Seventh	15:8
P8	Octave	2:1

Figure 8-3. Interval ratios in just intonation

In Practice

- In music notation, write the fundamental pitch G1 and its harmonic series up to the 16th harmonic. Compare with Figure 8-2 and check for overlapping overtones.

9. The Interval of the Perfect Fifth and the Circle of Fifths

The Quintal Relationship

The interval of the perfect fifth (P5) has played a central role in the development of Western music theory. Within the tonal system grounded in the diatonic scale—from early chant through the late Romantic era and into contemporary popular music—composers have consistently relied on the quintal relationship (a spacing of a perfect fifth) between chords to reinforce tonal centers through the dominant (V) and tonic (I) chords, expressed most clearly in the V–I cadence.

For example, in the G-major scale, the primary triads—tonic (G major), subdominant (C major), and dominant (D major)—are built on the 1st, 4th, and 5th scale degrees, respectively (Figure 9-1).

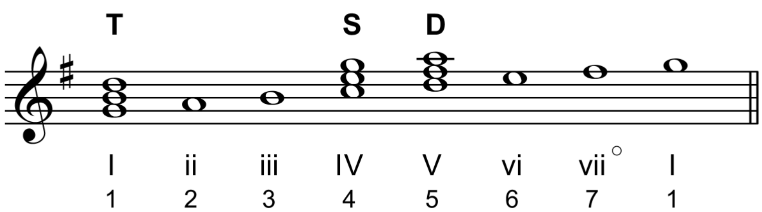


Figure 9-1. G-major scale and its primary triads: the tonic, subdominant, and dominant

Notably, both the dominant and subdominant triads stand in a quintal relationship to the tonic (each a perfect fifth away): the dominant lies a perfect fifth above, while the subdominant lies a perfect fifth below (Figure 9-2). This symmetrical configuration reinforces the structural and acoustic prominence of the perfect fifth within tonal harmony.

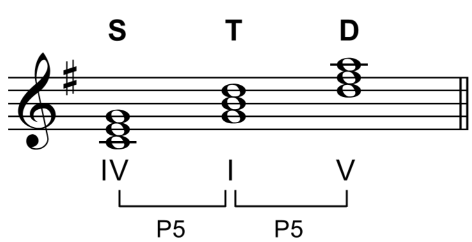


Figure 9-2. The quintal relationship of the dominant and the subdominant with the tonic

The interval of the perfect fifth (P5) has shaped much of Western music, not only in localized cadences within phrases and short segments, but also as a driving force in the structural design of large-scale forms such as the fugue, sonata, concerto, and symphony—particularly through its role in modulation.

The Circle of Fifths

Because of the structural prominence of the perfect fifth (P5) and the quintal relationships that shape tonal organization, music theorists developed a reference tool now known as the circle of fifths. Its earliest known example appears in 1677 in Nikolay Diletsky's *Grammatika musikiyskago peniya* (A Grammar of Musical Singing), written by a composer and theorist born in Kyiv, then part of the Polish-Lithuanian Commonwealth, and active in Muscovy. His treatise, intended to introduce Western-style composition to Eastern Orthodox musicians, used the circle to visualize key relationships and pathways of modulation.

Johann David Heinichen, a German Baroque composer and theorist, introduced an early version of the circle in his 1711 treatise *Neu erfundene und gründliche Anweisung* (A Newly Devised and Thorough Method), which he called the 'Musical Circle.' He returned to and refined the concept in his landmark 1728 treatise *Der General-Bass in der Composition* (The Thoroughbass in Composition), producing the more developed and modern-looking diagram most commonly associated with his name. His diagram presents the keys arranged by perfect fifths, along with their associated sharps and flats (Figure 9-3), and later versions also incorporate relative minor keys.

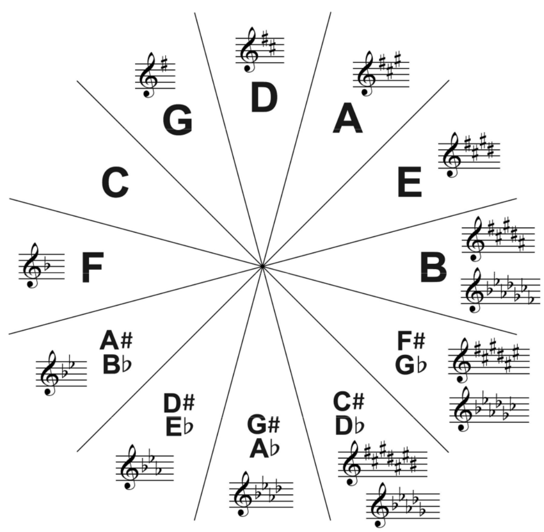


Figure 9-3. The traditional circle of fifths with sharps and flats

In Practice

- In music notation, write out perfect fifths (P5), in both directions (up and down) from the following notes: B♭, C#, D, A♭, E.
- In music notation, write out primary triads (tonic, subdominant, and dominant) for D-major scale, A♭-major scale, and B-major scale.
- Learn to quickly draw a circle of fifths from memory, only the main notes, F, C, G, D, A, E, B, F#, etc. as illustrated in Figure 9-3.

10. The Role of the Circle of Fifths in the Harmonic Processions

The Traditional Function of the Circle of Fifths

As mentioned in the previous chapter, the circle of fifths has traditionally been used to understand key signatures, visualize modulation paths, build chord progressions, and explore harmonic relationships among diatonic major and minor scales. Historically, each node on the circle represented a tonality: for example, F indicated the key of F-major with one flat, while E indicated the key of E-major with four sharps.

The Circle of Fifths as a Tool for Examining Sets

The theory of Harmonic Processions approaches the circle of fifths from a different perspective. In this framework, the circle is used exclusively to illustrate relationships between individual notes. Thus, F on the circle refers simply to the note F (in any register), and E refers to the note E (in any register). At no point will we use the circle to determine the number of accidentals in a key.

Let us consider the following examples: Figure 10-1 illustrates all notes of the C-major scale and Figure 10-2, the D-major 7th chord. Notice we simply circled the notes belonging to each set.

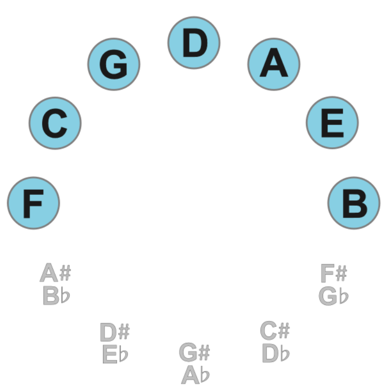


Figure 10-1. C-major scale, 42S (F 7-35), on the circle of fifths

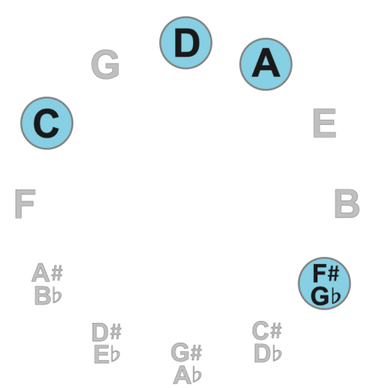


Figure 10-2. D-major 7th chord, 29# (F 4-27B), on the circle of fifths

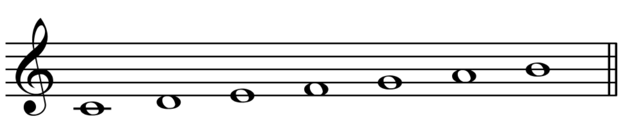


Figure 10-3. C-major scale, 42S (F 7-35), in music notation

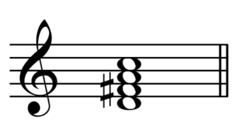


Figure 10-4. D-major 7th chord, 29# (F 4-27B), in music notation

Let us map two more examples (Figure 10-5 through Figure 10-8):

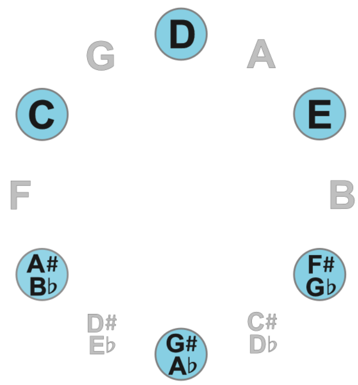


Figure 10-5. Whole-tone scale, 197S (F 6-35), on the circle of fifths

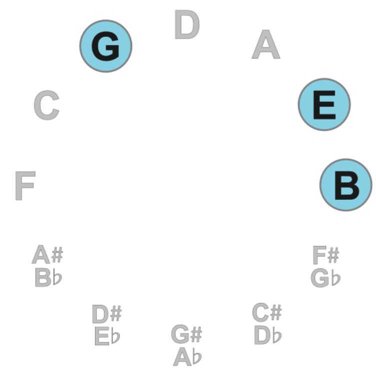


Figure 10-6. E-minor triad, 8♭ (F 3-11A), on the circle of fifths

Figure 10-7. Whole-tone scale, 197S (F 6-35), in music notation

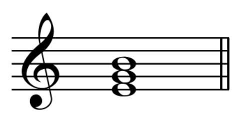


Figure 10-8. E-minor triad, 8♭ (F 3-11A), in music notation

As we introduce the theory of Harmonic Processions, certain patterns in the construction of sets will begin to emerge—patterns that express harmonic qualities and relationships. We will refer to the circle of fifths frequently to compare these patterns. Although quick sketches are practical and immediate, it may be worthwhile to invest in a physical tool to support one's harmonic explorations. One option is to use a round watercolor mixing dish with twelve slots, label each slot with a sticker, and use glass marbles to mark the relevant positions (Figure 10-9).

Circle of Fifths as the Pathway of the Harmonic Processions

We can already see that, within the theory of Harmonic Processions, the circle of fifths serves as the fundamental scaffolding on which all sonorities are constructed and examined. As our exposition unfolds, however, it will become clear that the circle of fifths is far more than a simple reference tool. It is the very pathway—and indeed the generative engine—behind the unfolding of Harmonic Processions. This mechanism will be explored in detail in Chapter 22.

Figure 10-9. A watercolor mixing dish as a circle-of-fifths tool

In Practice

- Draw the following on the circle of fifths:
  - A-major scale
  - G-minor triad
  - F-major 7th chord
  - Interval C–C#

11. The Triadic Form and Transposition

The Triadic Form

There are twelve major triads (trichords) within the circle of fifths, each one built on a successive note in the cycle of perfect fifths (P5). They are shown in Figure 11-1:

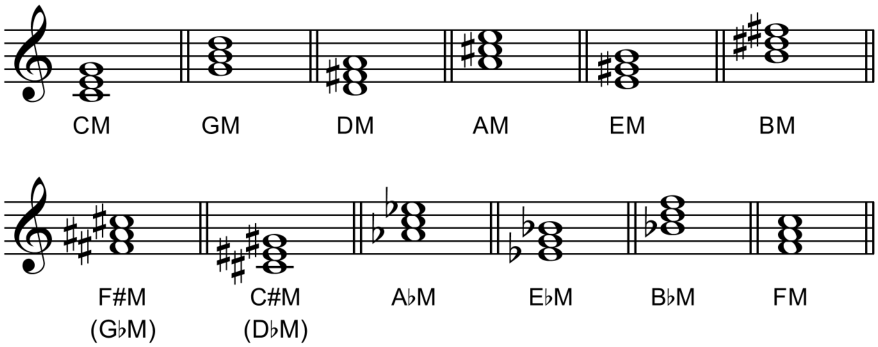


Figure 11-1. Major triads 8# (F 3-11B) built on the succession of perfect fifths

And this is how several of these triads map onto the circle of fifths (Figure 11-2):

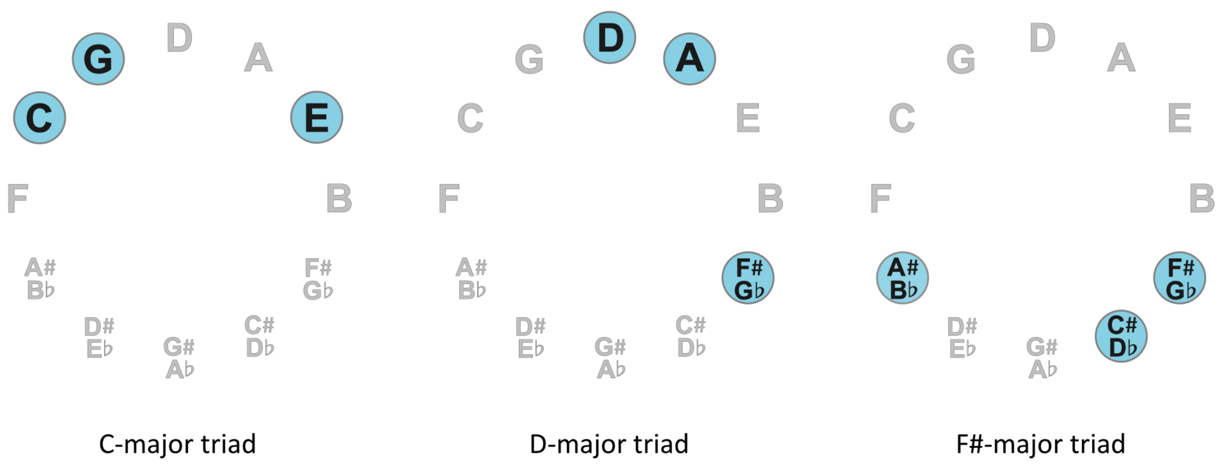


Figure 11-2. Selected major triads, 8# (F 3-11B), depicted on the circle of fifths

Notice the basic shape of the major triad. In the C-major triad, two notes cluster together to outline the perfect fifth (C–G), while the remaining note (E) lies three nodes away in the clockwise direction. The same configuration appears in the D-major triad: two adjacent notes forming the perfect fifth (D–A), with the third note (F#) positioned three steps farther along the circle. This enduring configuration is the characteristic shape of every major triad.

Now let us consider the shape of a minor triad 8♭ (F 3-11A). Its configuration resembles that of the major triad, except that it is reversed. In the examples below (Figure 11-3 and Figure 11-4), we observe two notes clustering together to outline the perfect fifth (E–B in the E-minor triad and G#–D# in the G#-minor triad), with the remaining note (G in the E-minor triad and B in the G#-minor triad) positioned three spaces away in counterclockwise direction.

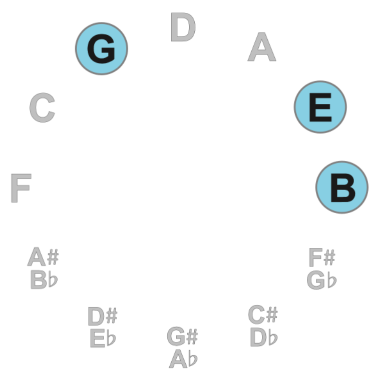


Figure 11-3. E-minor triad 8♭ (F 3-11A)
on the circle of fifths

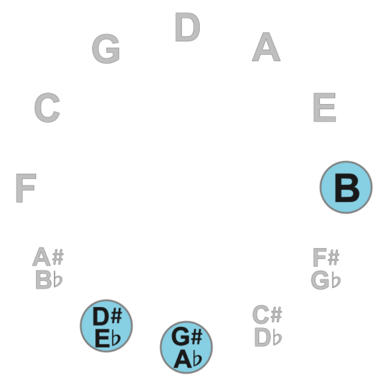


Figure 11-4. G#-minor triad 8♭ (F 3-11A)
on the circle of fifths

Transposition

We can construct any major or minor trichord by rotating the basic triadic form around the circle of fifths—a process commonly understood as transposition. Figure 11-5 illustrates this clearly: a clockwise rotation of the B♭-major triad by five nodes yields the A-major triad. Although these examples focus on major and minor triads, the same rotational mechanism applies to any set, regardless of its specific form.

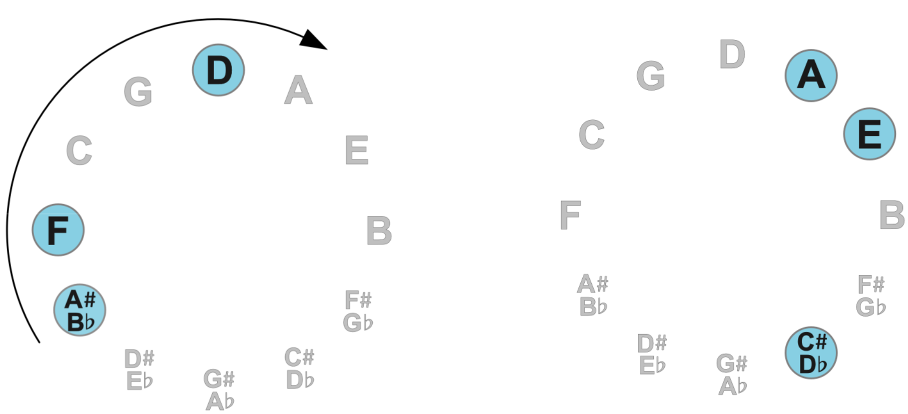


Figure 11-5. Clockwise transposition of the B♭-major triad yielding the A-major triad

In Practice

- On the circle of fifths draw the following sets:
  - C#-major 7th chord
  - E♭-major scale
  - The interval D–D#

- Transpose each set by three perfect fifths in the clockwise direction and draw the resulting transpositions on the circle of fifths.

12. The Row of Fifths

If the circle of fifths is cumbersome to draw—or if rotating sets makes one's head spin—we can always turn to the row of fifths instead. The row of fifths is simply a horizontal sequence of perfect fifths, and we can make each row shorter or longer depending on our needs.

Let us now revisit some of the sets we have already discussed, introduce a new one, and map them onto the row of fifths (Figure 12-1 through Figure 12-3).

Figure 12-1. The major triad, 8# (F 3-11B)

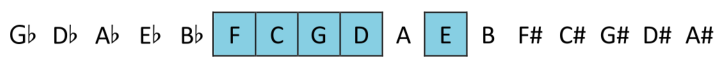


Figure 12-2. The Ionian pentachord, 19# (F 5-23B)

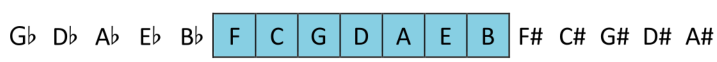


Figure 12-3. C-major scale, 42S (F 7-35)

While transposition on the circle of fifths is accomplished through rotation, on the row of fifths we simply slide the forms to the left or to the right. Let us revisit the example from the previous chapter and transpose the B♭-major triad to the A-major triad.

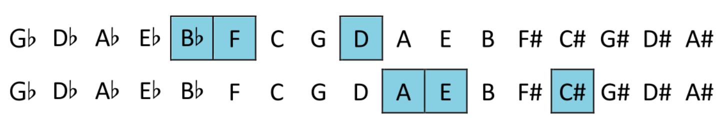


Figure 12-4. Transposition of the B♭-major triad to the A-major triad on the row of fifths

In Practice

- On the row of fifths, map the following sets:
  - G-minor triad
  - Whole-tone scale beginning on a note of your choice
  - A♭-major 7th chord

- On the row of fifths, transpose all of the sets you have just mapped, down by four perfect fifths.

13. Consonance and Dissonance

If the theory of music operated in a binary world, we could say that consonant intervals have a pleasing sound, while dissonant intervals feel unsettling, and leave it at that. But the situation is not quite that straightforward. For one, the perception of consonance and dissonance is partly subjective and shaped by personal taste. An interval may sound more or less consonant depending on the register in which it is played, the instrument that produces it, or its melodic and harmonic context. A listener's cultural background also plays a significant role in how dissonance is perceived. Music theorists generally agree that the perfect fifth (P5) is more consonant than the tritone (TT), but beyond that, there is no true consensus about the classification of the remaining intervals.

Historical Approach to Consonance and Dissonance

In antiquity, Pythagoras[5]—and in the medieval period, Boethius[6] and Isidore of Seville[7]—classified musical intervals according to simple numerical ratios derived from string lengths or frequencies, and understood intervals in the following way (Figure 13-1):

Interval	Classification
P8 (2:1 ratio)	Consonance
P5 (3:2 ratio)	Consonance
P4 (4:3 ratio)	Consonance
All others	Dissonance

Figure 13-1. Classification of intervals by Pythagoras, Boethius, and Isidore of Seville

Ptolemy[8] (Antiquity) recognized major thirds (M3) as potentially consonant and favored just-intonation systems that incorporated the 5:4 and 6:5 ratios. His views were later revived and expanded in the Renaissance by Gioseffo Zarlino[9] and, subsequently, by Johann Joseph Fux[10] (Figure 13-2).

In the Romantic era (roughly 1800–1850), interval classification became less rigidly codified than in earlier periods. Rather than being governed strictly by contrapuntal rules, intervals were increasingly valued for their expressive potential, color, and emotional resonance. As chromaticism intensified, composers ventured into new harmonic territories, paving the way for expressionism, modernism, and eventually post-tonalism, whose central aim was the liberation of dissonance from traditional constraints.

[5] Andrew Barker, *Greek Musical Writings*, vol. 1 (Cambridge: Cambridge University Press, 1984).
[6] Boethius, *Fundamentals of Music*, trans. Calvin M. Bower (New Haven: Yale University Press, 1989).
[7] Isidore of Seville, *The Etymologies*, trans. Stephen A. Barney et al. (Cambridge: Cambridge University Press, 2006).
[8] Ptolemy, *Harmonics*, trans. Andrew Barker (Cambridge: Cambridge University Press, 1989).
[9] Gioseffo Zarlino, *The Art of Counterpoint: Part Three of Le istitutioni harmoniche*, 1558, trans. Guy A. Marco and Claude V. Palisca (New Haven: Yale University Press, 1968).
[10] Johann Joseph Fux, *The Study of Counterpoint from Gradus ad Parnassum*, trans. Alfred Mann (New York: W. W. Norton, 1965).

Interval	Classification
U (Unison), P8, P5	Perfect Consonance
M3, m3, M6, m6	Imperfect Consonance
M2, m2, M7, m7, TT	Dissonance

Figure 13-2. Classification intervals by Ptolemy, Zarlino, and Fux

Below are a few modern and contemporary approaches to consonance, dissonance, and the classification of intervals. According to Paul Hindemith[11], every interval possesses a degree of harmonic force that is inversely proportional to its melodic force. In his system, an interval's harmonic force corresponds to its position within the harmonic series (Figure 13-3).

Interval	Harmonic Strength	Melodic Strength
P8	None	None
P5, P4	Strong	Minimal
M3, m6	Strongest	Less than m3, M6
m3, M6	Less than M3, m6	Less than M2, m7
M2, m7	Less than m3, M6	Strongest
m2, M7	Minimal	Strong
TT	No harmonic strength unless in the context of other notes	No melodic strength unless in the context of other notes

Figure 13-3. Classification of intervals by Hindemith

George Russell[12], treats the tritone above the tonic as a consonant interval because it derives from the Lydian dominant thirteenth chord.

Richard Bobbitt[13] divides intervals into four groups according to the density of the superimposed periodicities of their wave phases (Figure 13-4):

Interval	Density
U (Unison), P8, P5, P4	Low Density
M6, M3, m3, m6	Medium Density
m7, M2, M7, m2	High Density
TT	Special Case

Figure 13-4. Classification of intervals by Bobbitt

[11] Paul Hindemith, *The Craft of Musical Composition, Book I: Theory*, trans. Arthur Mendel and Otto Ortmann (Mainz: Schott, 1942).
[12] George Russell, *The Lydian Chromatic Concept of Tonal Organization* (Boston: Concept Publishing, 1953).
[13] Richard Bobbitt, "The Physical Basis of Intervallic Quality and Its Application to the Problem of Dissonance," *Journal of Music Theory* 3, no. 2 (1959): 224–245.

Vincent Persichetti[14] classifies the perception of intervals according to their position in the overtone series, as follows (Figure 13-5):

Interval	Classification
U (Unison), P5, P8	Open Consonance
M3, m3, M6, m6	Soft Consonance
P4	Consonance or Dissonance
TT	Ambiguous, Neutral, or Restless
M2, m7	Mild Dissonance
m2, M7	Sharp Dissonance

Figure 13-5. Classification of intervals by Persichetti

Dan Haerle[15] describes the minor 9th as the most dissonant interval, more dissonant than the minor second (even though most scholars consider them octave-equivalent), and classifies the tritone as slightly less consonant than the perfect fourth and perfect fifth.

Miguel A. Roig-Francoli[16], like Persichetti, classifies intervals according to their position in the overtone series, but adopts a different approach to the classification of the tritone (TT), as follows (Figure 13-6):

Interval	Classification
U (Unison) P8, P5, P4	Perfect Consonance
M3, m3, m6, M6	Imperfect Consonance
P4	Consonance or Dissonance, depending on context
m2, M2, m7, M7, TT	Dissonance

Figure 13-6. Classification of intervals by Roig-Francoli

The Traditional Approaches and the Theory of Harmonic Processions

The above examples are not exhaustive; they offer only a cursory overview of the ever-evolving theories and perceptions of consonance and dissonance. Although these approaches draw on intervallic ratios, the overtone series, and harmonic context or voicing, they differ significantly in how they classify intervals and interval classes. While it is not the aim of the present publication to diminish their importance in the development of music theory, we present them here as a contrasting backdrop to the theory of Harmonic Processions.

The theory of Harmonic Processions grounds the concepts of consonance and dissonance not in intervallic ratios, the overtone series, or harmonic context or voicing, but exclusively in the circle of fifths. One might argue that the circle of fifths itself arises from the overtone series and intervallic ratios, leaving little room for an alternative approach. However, as we introduce the theory, we will show that consonance and dissonance can be understood in a new light.

---

[14] Vincent Persichetti, *Twentieth-Century Harmony: Creative Aspects and Practice* (New York: W. W. Norton, 1961), 14.
[15] Dan Haerle, *The Jazz Language: A Theory Text for Jazz Composition and Improvisation* (Van Nuys, CA: Alfred Publishing, 1980).
[16] Miguel A. Roig-Francolí, *Harmony in Context* (New York: McGraw-Hill, 2003).

In Practice

- Play a variety of intervals on an instrument and construct a table ranking them from most consonant to most dissonant. Do not rely on theoretical writings; focus solely on your own subjective perception.

14. The Magnetism of Consonance and the Repulsion of Dissonance

The Geography of the Circle of Fifths

As we explore the geography of the circle of fifths, we observe its north–south and west–east polarities. Every note on the circle has an opposite, located exactly a tritone (TT) away—for example, F–B, C–F#, or D–G#. This tritone span, which forms the diameter of the circle, represents the greatest possible distance between two notes on the circle of fifths and corresponds to the largest interval class (Figure 14-1).

The diatonic notes—the white keys on the piano—occupy the northern hemisphere of the circle, while the chromatic notes—the black keys—congregate on the southern side. We can, of course, rotate the circle of fifths in other ways, but for the sake of continuity we will maintain this north–south orientation (Figure 14-2).

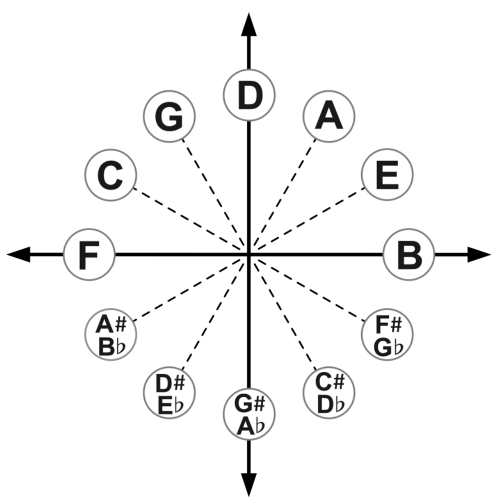


Figure 14-1. Polarity and symmetry of the circle of fifths

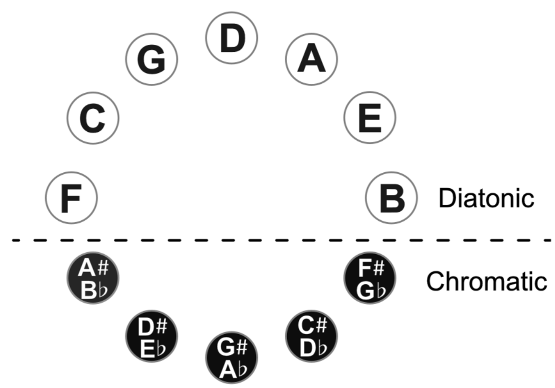


Figure 14-2. Diatonicism and chromaticism of the circle of fifths

Magnetism of Consonance

Let us acknowledge the magnetism characteristic of consonance. For example, when we map the pentachord C–D–E–G–A—also known as the pentatonic scale (Figure 14-3)—onto the circle of fifths, we find that this undeniably consonant set forms a compact cluster of four adjacent perfect fifths: C–G, G–D, D–A, and A–E (Figure 14-4).

This phenomenon of magnetism in the clustering of notes is directly related to consonance. In other words, notes that lie close together—adjacent on the circle of fifths—form more consonant sets than notes that are spaced farther apart around the perimeter.

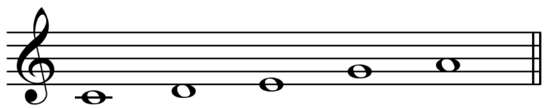


Figure 14-3. The pentatonic scale, 12S (F 5-35), in music notation

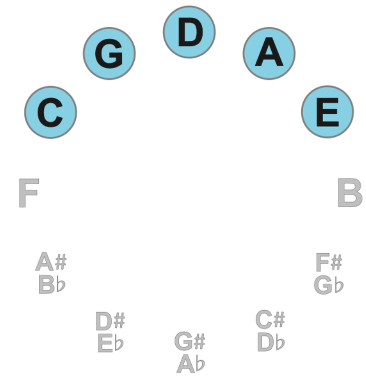


Figure 14-4. The pentatonic scale, 12S (F 5-35), mapped on the circle of fifths

Now, let us expand the previously mentioned pentatonic set C–D–E–G–A, by adding two notes, F and B, to form the diatonic set C–D–E–F–G–A–B (Figure 14-5 and Figure 14-6).

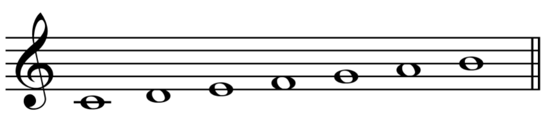


Figure 14-5. The diatonic set, 42S (F 7-35), in music notation

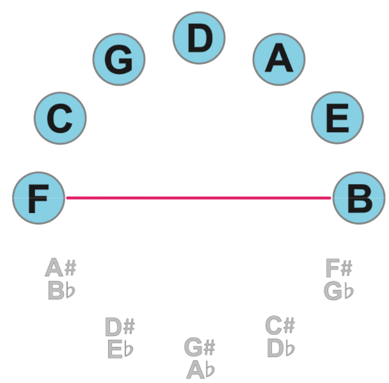


Figure 14-6. The diatonic set, 42S (F 7-35), mapped on the circle of fifths, with the tritone axis

Notice that we still maintain the cluster of perfect fifths (P5), but this time it is wide enough to encompass the tritone (TT) F–B. And while the pentatonic scale sounds neutral and mild, it is the addition of this single tritone that alters the set's character and gives it its unmistakable diatonic modality. The tritone axis on the circle of fifths is the final frontier between traditional diatonic tonality and chromaticism.

Repulsion of Dissonance

The tritone, embodying two opposite poles, carries a great deal of tension, and its two notes exist in a state of magnetic repulsion—or dispersion—characteristic of dissonance. Let us observe this tension by comparing the pentatonic scale with two highly dissonant pentachords: 68# (F 5-7A) and 103S (F 5-33) (Figure 14-7 through Figure 14-10).

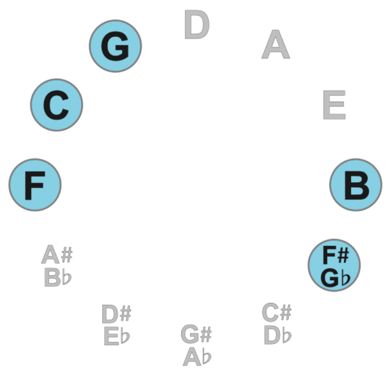


Figure 14-7. Pentachord 68# (F 5-7A)

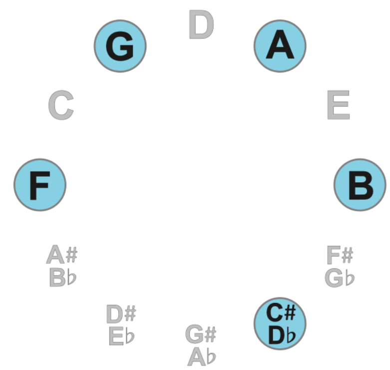
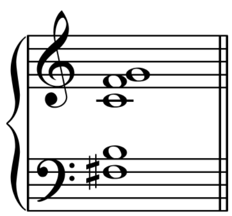
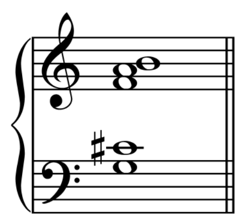


Figure 14-8. Pentachord 103S (F 5-33)



68#

Figure 14-9. Pentachord 68# (F 5-7A)



103S

Figure 14-10. Pentachord 103S (F 5-33)

How do the forms of these dissonant pentachords differ from the pentatonic? Do you observe clustering and magnetism, repulsion, or both? Which pentachord exhibits greater dissonance, and why? The answers to these questions will be explored in the chapters that follow.

In Practice

- If a composition is based on the pentatonic set C–D–E–G–A and the composer wishes to startle the audience with a dramatic harmonic shift, which notes might she or he choose? Map them on the circle of fifths.
- On the circle of fifths, map several chords or scales with which you are familiar. Do you observe clustering and magnetism, repulsion, or both? Are the resulting collections consonant or dissonant?

15. The Resolution of the Tritone

The Historical Approach to the Resolution of the Tritone

The awkward interval of the tritone, spanning three consecutive whole tones, was known to medieval musicians as the *Devil in music*, and its instability was to be avoided at all costs. As stylistic preferences evolved, however, the tritone became a powerful tool for creating harmonic tension. Historically, this tension was always resolved through stepwise motion—either outward to a minor sixth (m6) or inward to a major third (M3) (Figure 15-1 and Figure 15-2).

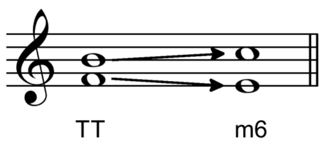


Figure 15-1. Resolution of the tritone outward to m6

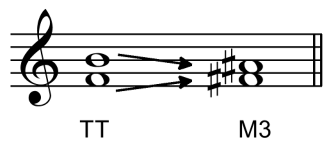


Figure 15-2. Resolution of the tritone inward to M3

The historical approach to harmony maintains that the instability of the tritone longs for resolution to a consonant interval, largely because it lies between two consonant intervals—a major third and a minor sixth—each only a half step away. We are taught that it is the power of the leading tones, or voice leading, that creates this inevitable pull toward consonance—a kind of gravitational attraction, as it is sometimes described.

But is this truly the primary force? For the tritone to resolve, the half-step motion must proceed in contrary directions: one note must move downward and the other upward. Otherwise, the result is simply another tritone, displaced by a half step (Figure 15-3).

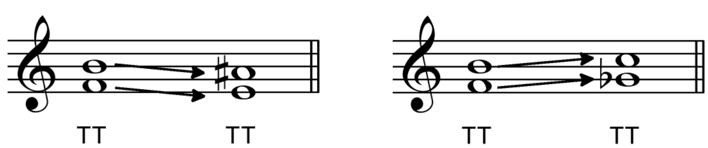


Figure 15-3. Half-step motion of the tritone without a resolution

Do the so-called leading tones possess such power that they not only propel the tritone toward resolution but also communicate with one another to coordinate their direction? Of course not. Their motion is determined by the harmonic context and, more fundamentally, by the magnetism of the circle of fifths.

The Resolution of the Tritone on the Circle of Fifths

Once we map the resolution of the tritone onto the circle of fifths, we see clearly that it is predicated on the consonant magnetism of the circle's poles, and that the tritone axis itself possesses a repelling, dissonant quality that demands movement toward either pole (Figure 15-4 and Figure 15-5).

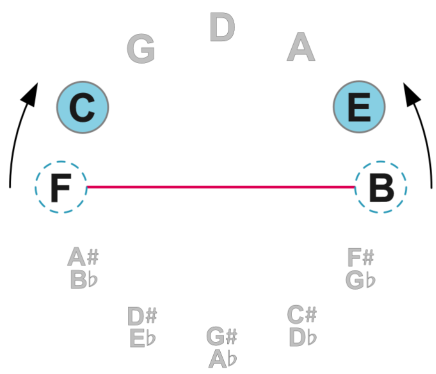


Figure 15-4. Resolution of the tritone in the northern hemisphere

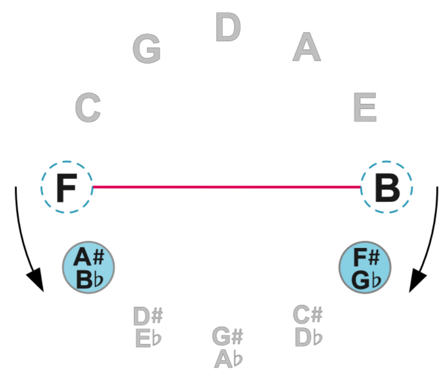


Figure 15-5. Resolution of the tritone in the southern hemisphere

In this model, the resolution of the tritone is not driven primarily by half-step voice leading (the traditional pull of leading tones toward the tonic) but rather by migration of perfect fifths: F is replaced by C (or B♭), and B is replaced by E (or F#), drawn by the magnetic pull toward the consonance of the northern or southern hemisphere and away from the repulsion of the tritone axis (F–B).

The Oblique Resolution of the Tritone

Given that the resolution of the tritone is predicated on the magnetism of consonance and the repulsion of dissonance, as illustrated on the circle of fifths, we can extrapolate that although the tritone most readily resolves to the neighboring fifths on the circle, it can also resolve obliquely. This oblique resolution is achieved not through motion to the adjacent perfect fifths but to any other notes within the same hemisphere (Figure 15-6). Below are several examples of such oblique resolutions of the tritone (Figure 15-7)

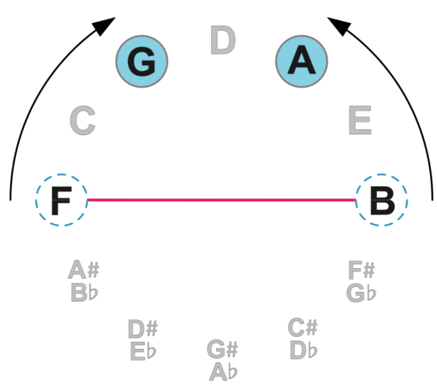


Figure 15-6. The oblique resolution of the tritone on the circle of fifths

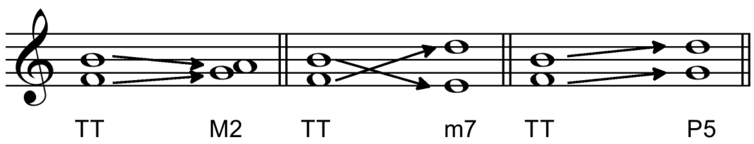


Figure 15-7. Examples of oblique resolutions of the tritone F–B

In Practice

- Write out the traditional resolutions of the tritone between D and G#, and then between E♭ and A, into both hemispheres. Next, write oblique resolutions to those same hemispheres. Map all resolutions on the circle of fifths and notate them on a staff.

16. Sharp and Flat Projections

The Core and the Projection

In Chapter 11, we introduced the triadic form of the major and minor triads. Let us now explore this concept further by examining the major triad, 8# (F 3-11B) and the Lydian pentachord, 33# (F 5-24B), mapped on the circle of fifths (Figure 16-1 and Figure 16-2).

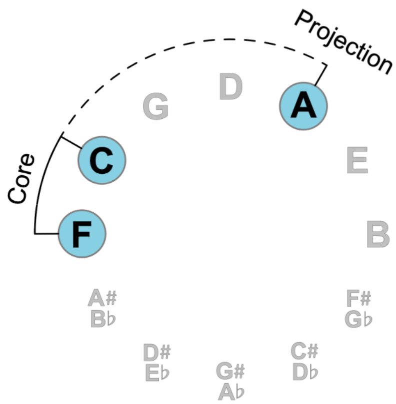


Figure 16-1. The major triad, 8# (F 3-11B)

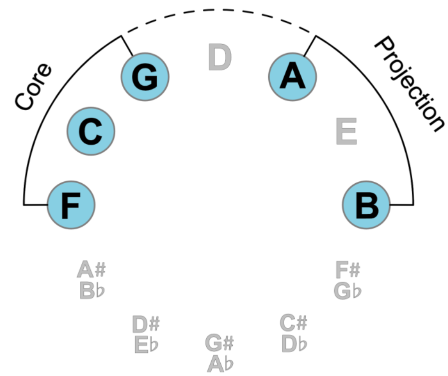


Figure 16-2. The Lydian pentachord, 33# (F 5-24B)

Let us take a closer look at the shapes of these chords. Both sets are asymmetrical, with the clusters of perfect fifths congregating on one side and skewing the centers of gravity to the left. Detached from these clusters are one note (A) in the case of the triad and two notes (A and B) in the case of the pentachord, projecting to the right.

This type of formation is reminiscent of the Big Dipper (Figure 16-3), a star pattern within the constellation Ursa Major (the Great Bear). For millennia, the Big Dipper—together with the Little Dipper—has guided navigators toward Polaris, the North Star. The formation is shaped like a dipper or bowl with a handle and is sometimes described as a ladle.

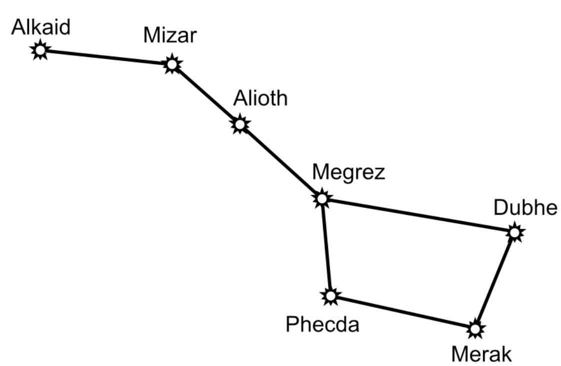


Figure 16-3. The Big Dipper

Similarly, in the major triad and the Lydian pentachord, we can identify the core with its center of gravity—the cluster of notes (equivalent to the bowl)—and a projection (a handle) extending to one side, which in our sets points clockwise.

This basic pattern—a core and a projection, oriented either clockwise or counterclockwise—appears in the majority of sets and plays a vital role in classifying their harmonic characteristics.

The Sharp Projection

When a set's projection points in the clockwise direction, the set is described as sharp-projecting. Both the major triad 8# and the Lydian pentachord #33 in the preceding examples are sharp-projecting. We can also say that they project toward the sharps or that they have sharp projections. The Harmonic-Processions numbers of sharp-projecting sets are marked with the sharp sign (#). Examine the Lydian pentachord 33# (F 5-24B) again in Figure 16-4.

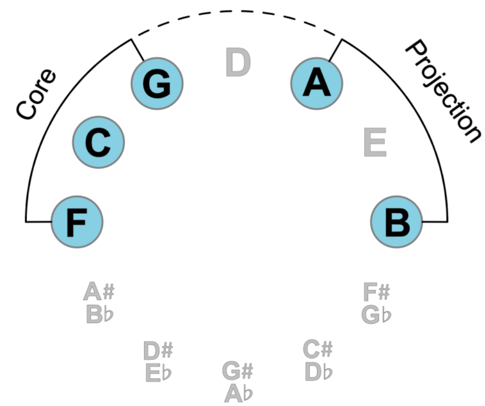


Figure 16-4. The sharp-projecting Lydian pentachord, 33# (F 5-24B)

The Flat Projection

When a set's projection points in the counterclockwise direction, the set is described as flat-projecting. We can also say that it projects toward the flats or that it has a flat projection. The Harmonic-Processions numbers of flat-projecting sets are marked with the flat sign (♭). Examine the Hirajōshi scale, 38♭ (F 5-20B), in Figure 16-5.

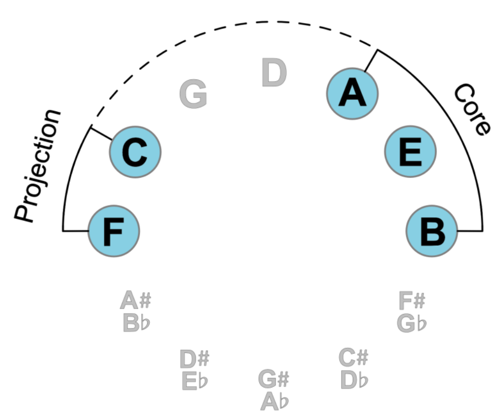


Figure 16-5. The flat-projecting Hirajōshi scale, 38♭ (F 5-20B)

The Modality Shift

Let us now experiment with the projection of the F-major triad (Figure 16-6 through Figure 16-8). We take the sharp-projecting note (A) and slide it to the opposite side of the core—three spaces apart, just as in its original form—so that it now projects toward the flats. What do we find? We obtain the flat-projecting triad of F-minor. By reversing the projection, we have reversed the modality of the set.

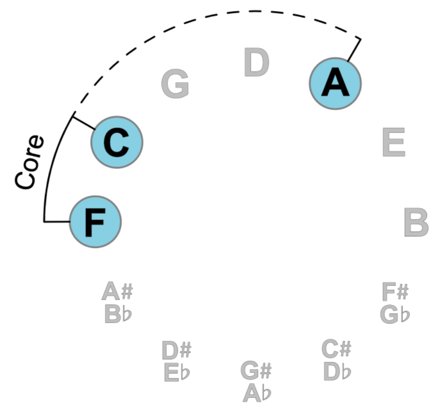


Figure 16-6. Sharp-projecting F-major triad, 8# (F 3-11B)

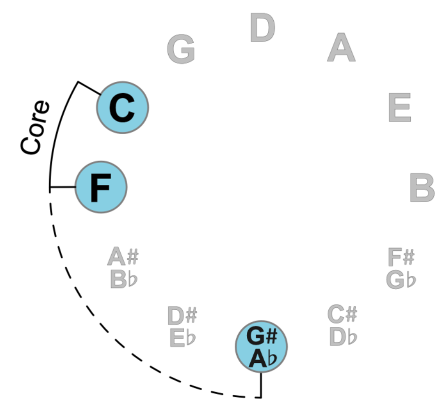


Figure 16-7. Flat-Projecting F-minor triad, 8♭ (F 3-11A)

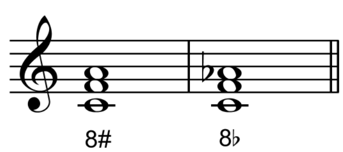


Figure 16-8. F-major and F-minor triad

The preceding experiment reveals an important principle: while the core of a set provides its harmonic foundation, the projection determines its modality[17]—its characteristic flavor.

Let us now consider the sharp-projecting Lydian pentachord and its flat-projecting counterpart, the Phrygian pentachord (Figure 16-9 through Figure 16-11). Once again, the core F–C–G remains in place while we slide the two sharp-projecting notes (A and B) so that they now project toward the flats (E♭ and D♭). Notice again the resulting change of modality between the two sonorities.

[17] Modalities are discussed in Chapter 26.

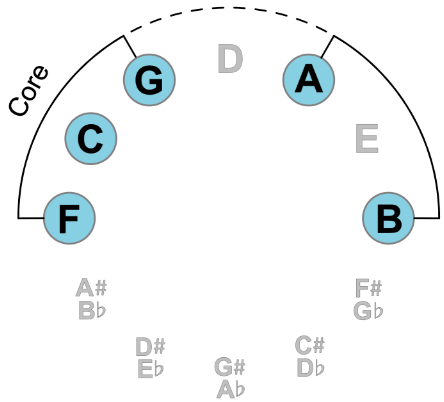


Figure 16-9. The sharp-projecting Lydian pentachord, 33# (F 5-24B)

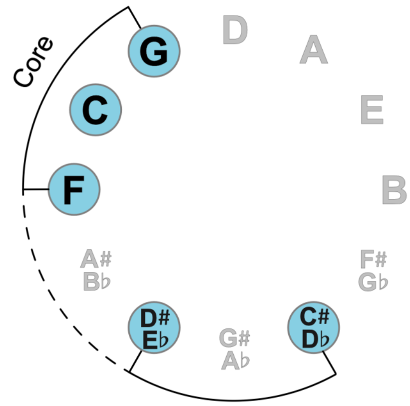


Figure 16-10. The flat-projecting Phrygian pentachord, 33♭ (F 5-24A)

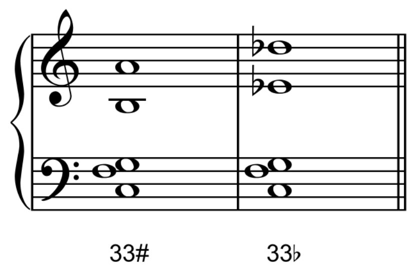


Figure 16-11. The Lydian pentachord, 33#, and the Phrygian pentachord, 33♭

In Practice

- On the circle of fifths, mark the following sets. Identify their cores and projections and determine whether each set is flat-projecting or sharp-projecting.
  - D♭–A♭–B♭–F–C
  - E–F#–C#–G#
  - F–C–D–A–B–F#–C#

- Examine the following tables:
  - **Table 5** - The Natural Harmonic Procession of Sharp-Projecting Sets
  - **Table 6** - The Natural Harmonic Procession of Flat-Projecting Sets

17. Span

Span defines the furthest point to which a set extends on the circle of fifths. For example, a major triad F–A–C—has a span of 5 (Figure 17-1) because from F to A on the circle, there are five notes in total. The diatonic set C–D–E–F–G–A–B—has a span of 7 (Figure 17-2).

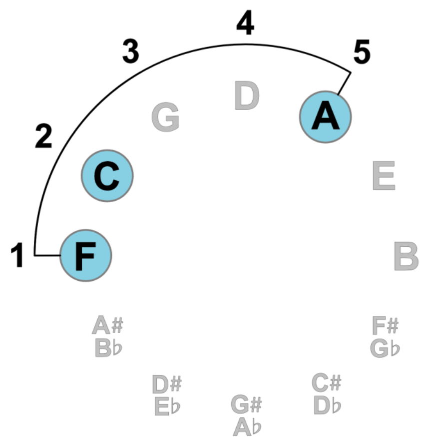


Figure 17-1. The span of the major triad, 8# (F 3-11B)

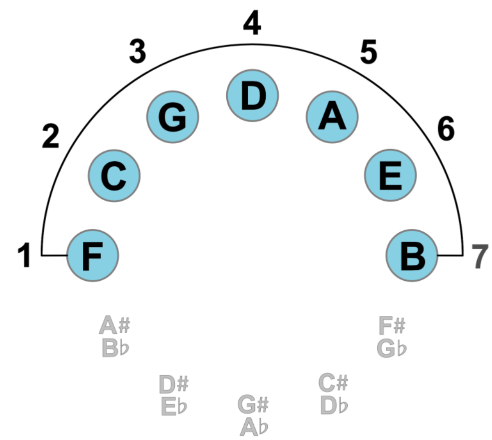


Figure 17-2. The span of the diatonic set, 42S (F 7-35)

Generally speaking, a greater span brings greater harmonic complexity and greater dissonance; however, it is not the sole determinant of a set's dissonance level. Span must be considered in combination with note content. A set with a larger note content will usually be more dissonant than one with a smaller note content, even if both share the same span.

Consider the following examples: the whole-tone scale, 197S (F 6-35), and the Northern Lights chord, 221S (F 11-1) (Figure 17-3 and Figure 17-4). Both have a span of 11, yet the latter is more complex and therefore more dissonant.

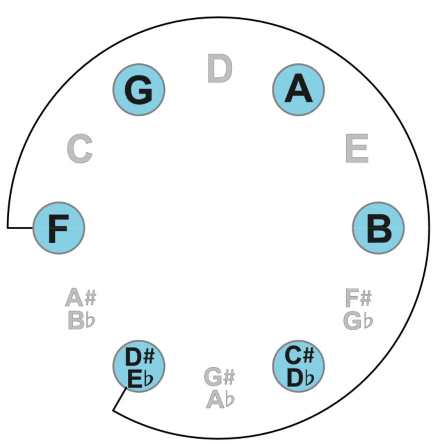


Figure 17-3. The whole-tone scale, 197S (F 6-35), span of 11

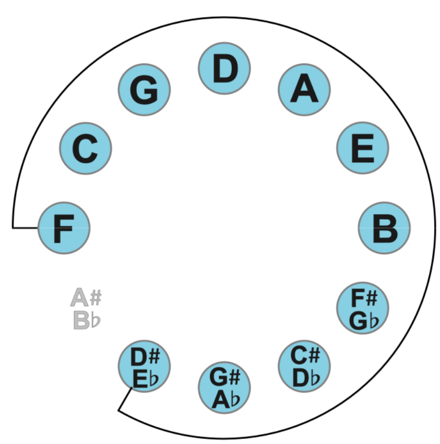


Figure 17-4. The Northern Lights chord, 221S (F 11-1), span of 11

Span plays a vital role in the sequence of the Natural Harmonic Procession (Chapter 22) and in the definition of modalities (Chapter 26).

In Practice

- In **Table 2** - The Natural Harmonic Procession of All Sets, locate the following sets and specify their spans:
  - Schoenberg Hexachord, 121#
  - Tristan chord, 29♭
  - Messiaen's 4th mode, 194S

- List as many sets as you know whose span is greater than 7

- Experiment with sets whose span is 5 or less. How many can you construct on your own?

18. The Quintal Prime Form

By now we have seen several sets mapped on the circle of fifths. In the case of the major triad, we have examined it in multiple transpositions (Chapter 11). With the growing variety of shapes and transpositions, it becomes increasingly difficult to compare different sonorities objectively. How do we identify differences and similarities between a triad and a tetrachord, or between a tetrachord and a hexachord? In order to compare all sets objectively—regardless of their transposition or tonality—we must consider them in their quintal prime form.

The Quintal Prime Form Defined

The quintal prime form is the ordering of a set's notes along the circle of fifths in the most compact possible configuration, beginning from the side (or end) that anchors the core and its center of gravity. The spelling of the quintal prime form includes as many diatonic (non-chromatic) notes—white keys on the piano—as the configuration allows; therefore, sharp-projecting sets begin on F, and flat-projecting sets begin on B.

When the quintal prime form is not immediately apparent—particularly in more complex sets—we first identify the widest empty gap between notes to determine the overall span of the set. We then examine both the clockwise and counterclockwise directions to find the configuration in which the projection contains the fewest notes and the core contains the most.

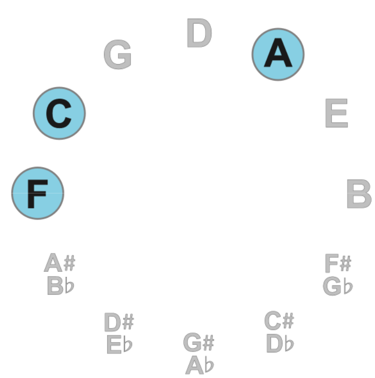


Figure 18-1. The quintal prime form of the major triad, 8# (F 3-11B)

The quintal prime form of the sharp-projecting major triad begins with F, the first diatonic note within the circle of fifths, and is spelled F–C–A, following the clockwise order of notes (Figure 18-1). Notice that the core and the center of gravity lie with the notes F and C, while A—the projection—stands alone, farther away.

Now let us compare the major triad with the Ionian pentachord C–G–D–A–B (Figure 18-2). When we map the pentachord on the circle of fifths, we see that it is not in its quintal prime form. But once we rotate it counterclockwise so that its core begins on F (Figure 18-3), we can objectively compare it with the preceding major trichord F–C–A.

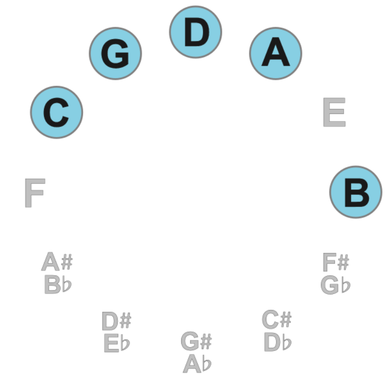


Figure 18-2. Ionian pentachord, 19# (F 5-23B), C–G–D–A–B

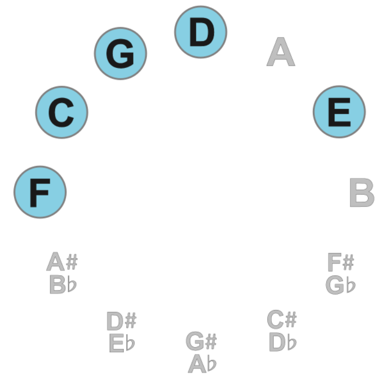


Figure 18-3. Ionian pentachord, 19# (F 5-23B) in quintal prime form, F–C–G–D–E

And here is the side-by-side comparison of the major triad and the Ionian pentachord, both in their quintal prime form (Figure 18-4).

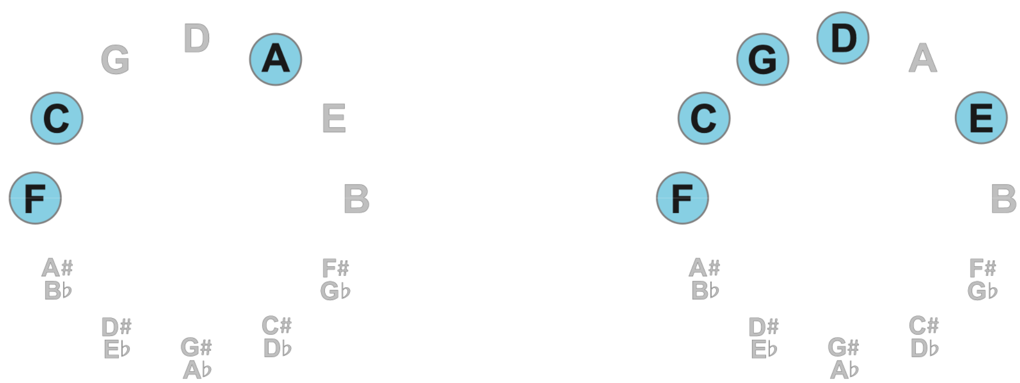


Figure 18-4. Major triad, 8# (F 3-11B), and the Ionian pentachord, 19# (F 5-23B), in quintal prime form, comparison

When comparing these two sets, we observe the following:

- Both are sharp-projecting.
- Both the major triad and the Ionian pentachord share two common tones (F and C).
- Both cores of the sets with their centers of gravity begin on F, but the pentachord's core is wider, forming a four-note cluster (F–C–G–D).
- Both the major triad and the Ionian pentachord have projections consisting of a single detached note: in the major triad, A—three steps clockwise from the core; in the Ionian pentachord, E—two steps clockwise from the core.
- The major triad has a span of 5, whereas the pentachord has a span of 6.

The Quintal Prime Form of Sharp-Projecting Sets

The quintal prime form of sharp-projecting sets always begins with F, the first diatonic note within the circle of fifths, and extends clockwise, following the order of notes toward the sharps. Below illustrated is the sharp-projecting Lydian augmented pentachord, 80# (F 5-30B), in its quintal prime form F–C–G–A–C# (Figure 18-5).

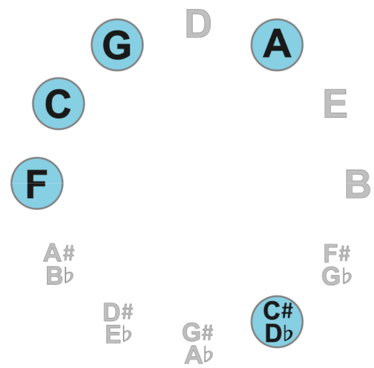


Figure 18-5. Sharp-projecting Lydian augmented pentachord, 80# (F 5-30B), in quintal prime form

The Quintal Prime Form of Flat-Projecting Sets

The quintal prime form of flat-projecting sets always begins with B, the last diatonic note within the circle of fifths, and extends counterclockwise, following the order of notes toward the flats. Below illustrated is the flat-projecting Hirajōshi scale, 38♭ (F 5-20B), in its quintal prime form B–E–A–C–F (Figure 18-6).

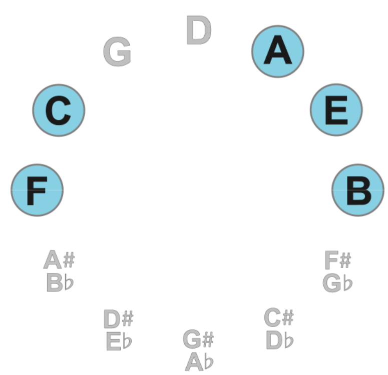


Figure 18-6. Flat-projecting Hirajōshi scale, 38♭ (F 5-20B), in quintal prime form

Let us map a few more examples of flat-projecting sets in their quintal prime form on the circle of fifths. Shown below are the previously discussed Phrygian pentachord, 33♭ (F 5-24A), (Figure 18-7) and the Sacher hexachord, 89♭ (F 6-Z11B), (Figure 18-8). Notice how their cores, along with their centers of gravity, skew to the right, while the detached notes project toward the flats.

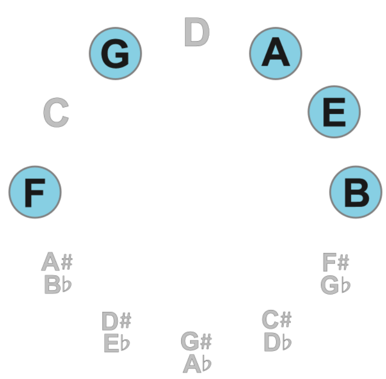


Figure 18-7. Phrygian pentachord, 33♭ (F 5-24A), in quintal prime form

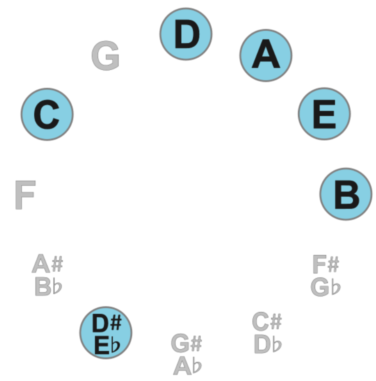


Figure 18-8. Sacher hexachord, 89♭ (F 6-Z11B), In quintal prime form

Sharps or Flats?

Let us revisit the sharp-projecting and flat-projecting designations. When in their quintal prime form, sets with clockwise projections will always project toward the sharps, while sets with counterclockwise projections will always project toward the flats. Although we could refer to these projections simply as clockwise and counterclockwise, the terms sharp-projecting and flat-projecting more clearly reflect the sets' associated modalities[18].

Is it possible for flat-projecting sets to contain sharps, or for sharp-projecting sets to contain flats? Of course. Any set can be transposed—rotated around the circle of fifths—into any key. However, when we examine the characteristics of a set as a type, we must always refer to its quintal prime form (beginning on F for sharp-projecting sets and on B for flat-projecting sets). In their quintal prime forms, sharp-projecting sets contain only sharps, while flat-projecting sets contain only flats.

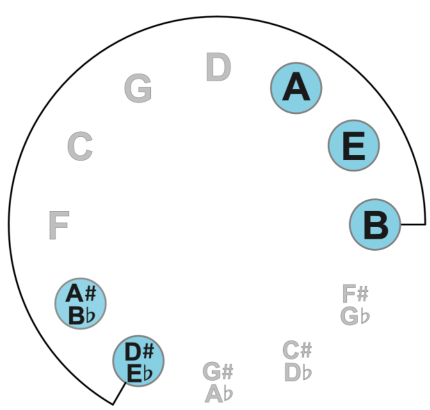


Figure 18-9. Pentachord 68# (F 5-7A), transposed

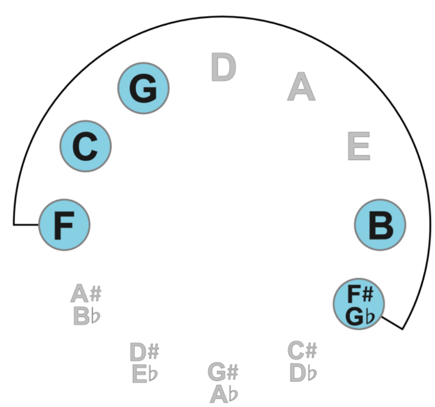


Figure 18-10. Pentachord, 68# (F 5-7A), in quintal prime form

[18] Modalities are discusses in Chapter 26.

Take a look at the set B–E–A–B♭–E♭, the pentachord, 68# (F 5-7A), in Figure 18-9. It contains flats—but is it a flat-projecting set? Once we identify its core, with the center of gravity at A–E–B, and its projection E♭–B♭, we recognize it as a sharp-projecting set. To find the pentachord's quintal prime form, we transpose it so that it begins on F, and observe that, whether transposed or not, it consistently projects clockwise. We therefore classify it as sharp-projecting, with only one chromatic note, F# (Figure 18-10), rather than the two flats E♭ and B♭.

When in doubt, it is helpful to analyze the span of a set and select the projection with the more compact span. Notice the span of 9 from B to E♭, moving counterclockwise in Figure 18-9, and the span of 8 from F to F#, moving clockwise in Figure 18-10.

In Practice

- Map the following sharp-projecting sets on the circle of fifths and determine their quintal prime forms by transposing each set to begin on F:
  - F#–A–A#–B
  - A–B♭–C

- Map the following flat-projecting sets on the circle of fifths and determine their quintal prime forms by transposing each set so that it begins on B:
  - B♭–C–C#–E
  - B–C–D–E

- Determine whether the following sets are sharp-projecting or flat-projecting and map each one on the circle of fifths in its quintal prime form:
  - E–G#–D#–C
  - F–D–A–E–F#–C#
  - E–F#–C#–A#

19. The Numeric Quintal Prime Form

While the quintal prime form consists of letters that represent the notes, the numeric quintal prime form expresses the same information using numeric values. This approach is especially useful for transposition and for quick comparison between sets.

Consider the Diminished Major 7th chord, 49# (F 4-18A), whose quintal prime form is F–C–A–F# (Figure 19-1). We can express this information numerically by measuring the span of each note relative to the first note, F. In this case, the numeric quintal prime form is 1,2,5,8. We mark it with the sharp sign (#1,2,5,8) to indicate that the set is sharp-projecting and that we are counting in the clockwise direction.

					1	**2**	3	4	**5**	6	7	**8**				
G♭	D♭	A♭	E♭	B♭	F	C	G	D	A	E	B	F#	C#	G#	D#	A#

Figure 19-1 Diminished Major 7th chord expressed as #1,2,5,8 built on F

Now we can use the numeric quintal prime form to transpose the pentachord around the circle of fifths while preserving the exact configuration of the set. Let us transpose this sonority so that it begins on D♭ (Figure 19-2).

					1	**2**	3	4	**5**	6	7	**8**				
D	A	E	C♭	G♭	D♭	A♭	E♭	B♭	F	C	G	D	A	E	B	F#

Figure 19-2 Diminished Major 7th chord expressed as #1,2,5,8 built on D♭

The use of the numeric quintal prime form not only speeds up the transposition process but also provides a harmonic overview of a musical passage. When we label sets with their numeric quintal prime forms, we can immediately measure their spans, identify potential common tones, define their modalities, compare their complexities, and determine the direction of their projections.

Let us compare the numeric quintal prime forms of the following sonorities:

HP	Forte	Name	Numeric Quintal Prime Form
19#	5-23B	Ionian pentachord	#1,2,3,4,6
122#	7-20A	Persian scale	#1,2,3,4,5,8,9
55♭	6-Z47B	6-note Blues scale	♭1,2,3,4,5,8

Figure 19-3 Numeric quintal prime form comparison

From the numeric quintal prime forms, we can deduce the following:

- We are comparing a pentachord (Ionian), a heptachord (Persian scale), and a hexachord (the 6-note Blues scale).
- We see the spans of 5 (Ionian pentachord), 9 (Persian scale) and 8 (6-note Blues).
- Two sets are sharp-projecting (Ionian and Persian) and one is flat-projecting (6-note Blues).
- All three sonorities contain a cluster of four adjacent perfect fifths (1,2,3,4) and two of them (Persian and the 6-note Blues scale) contain a cluster of five adjacent perfect fifths (1,2,3,4,5).
- The cluster of four adjacent perfect fifths 1,2,3,4 also appears as a cluster 2,3,4,5 in the Persian and the 6-note Blues scale
- The Persian scale, except for one note (9), appears to be a mirror of the 6-note Blues scale[19]

In Practice

- Examine the numeric quintal prime forms in **Table 3** - The Natural Harmonic Procession of All Sets, Including the Numeric Quintal and the Chromatic Prime Form.

[19] More on mirror sets in Chapter 21.

20. Symmetrical Sets

The Characteristics of Symmetrical Sets

Besides sharp-projecting and flat-projecting sets, we can also observe symmetrical sets. In these sets, the distinction between core and projection may be more ambiguous, while the center of gravity remains balanced at the midpoint. Symmetrical sets project toward both the sharps and the flats simultaneously.

Take a look at the following symmetrical sets: the dominant 9th chord, 35S (F 5-34), in Figure 20-1, and hexachord 111S (F 6-Z23) in Figure 20-2. The dashed lines across the diameter of each circle represent the lines of symmetry—a kind of mirror that divides the set into two identical, reflected segments.

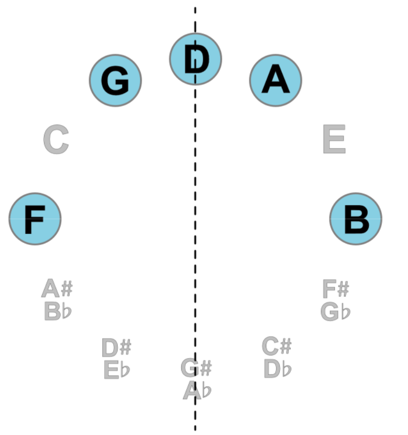


Figure 20-1. Dominant 9th chord, 35S (F 5-34)

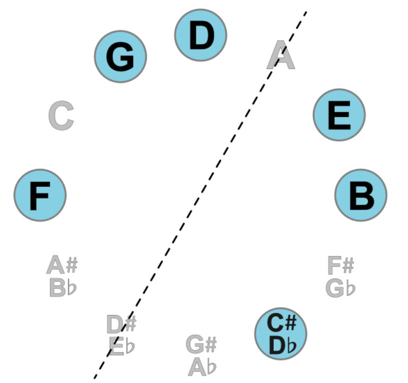


Figure 20-2. Hexachord 111S (F 6-Z23)

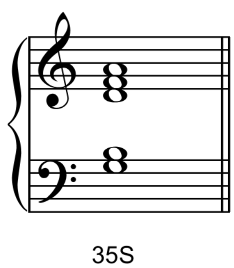


Figure 20-3. Dominant 9th chord, 35S (F 5-34),
in music notation

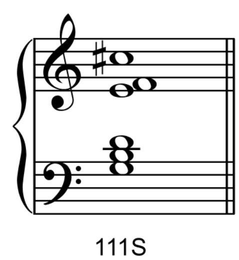


Figure 20-4. Hexachord 111S (F 6-Z23)
in music notation

The Quintal Prime Form of Symmetrical Sets

The quintal prime form of symmetrical sets may be notated either beginning on F, in clockwise direction, or beginning on B, in counterclockwise direction. In the Harmonic Processions tables, for the sake of clarity and to facilitate comparison with other sets, symmetrical sets are listed twice: once in the sharp-projecting quintal prime form and once in the flat-projecting quintal prime form.

Take a look at the pentatonic scale 12S (F 5-35) in Figure 20-5 and Figure 20-6. Its sharp-projecting quintal prime form is F–C–G–D–A, notated in the Harmonic Processions tables as 12#S, and its flat-projecting quintal prime form is B–E–A–D–G, notated as 12♭S. This is a singular symmetrical pentachord, marked with the letter S to signify its symmetry. The #/♭ designations apply only to its dual quintal prime forms when compared with other sharp- or flat-projecting sets in the tables. In general, however, when referring to symmetrical sets, we mark them only with the letter S (here, 12S), remembering that symmetry always implies the presence of both the sharp and the flat projection.

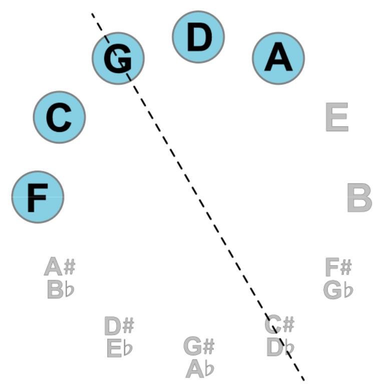


Figure 20-5. The pentatonic scale, 12S (F 5-35), in sharp-projecting quintal prime form

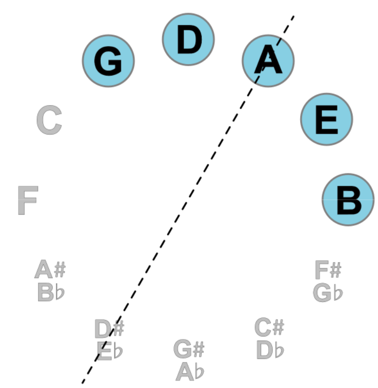


Figure 20-6. The pentatonic scale, 12S (F 5-35), In flat-projecting quintal prime form

The Numeric Quintal Prime Form of Symmetrical Sets

The numeric quintal prime forms of symmetrical sets do not require the sharp (#) or flat (♭) designation unless we want to specify the set's particular projection. For example, the symmetrical pentatonic scale illustrated in Figure 20-5 in its sharp-projecting form and in Figure 20-6 in its flat-projecting form has the numeric quintal prime forms #1,2,3,4,5 and ♭1,2,3,4,5 respectively. However, the #/♭ designation can be omitted, since symmetrical sets can be constructed with the same string of numbers in either direction. In the Harmonic Processions tables, because we present all symmetrical sets in both projections, the #/♭ designations are always included.

Lines of Symmetry

Symmetrical sets may have more than one line of symmetry, and these lines may pass either through the notes or between them, as shown in Figure 20-7 through Figure 20-10.

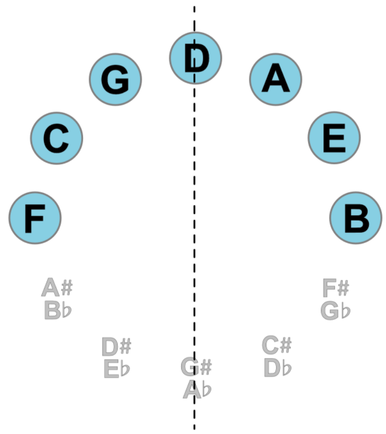


Figure 20-7. Diatonic set, 42S (F 7-35), with a single line of symmetry crossing through note D

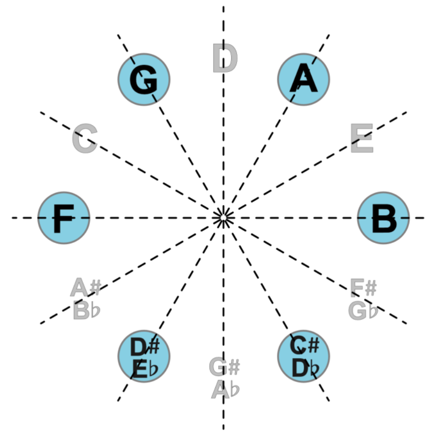


Figure 20-8. Whole-tone scale, 197S (F 6-35), with 6 lines of symmetry

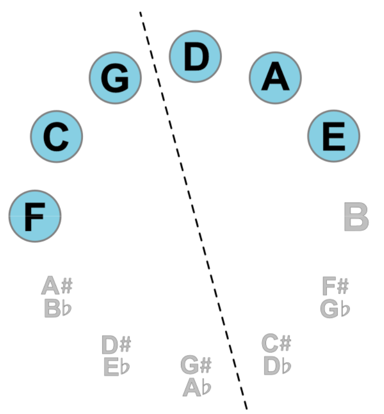


Figure 20-9. Ionian hexachord, 22S (F 6-32), with a single line of symmetry crossing between notes G and D

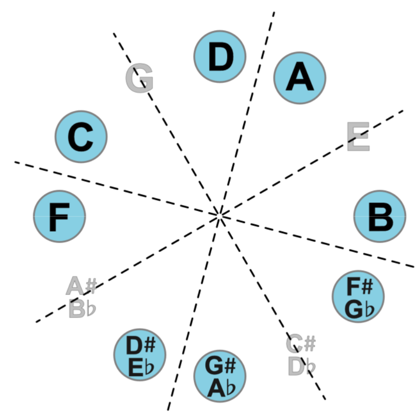


Figure 20-10. Octatonic scale, 213S (F 8-28), with 4 lines of symmetry crossing through and between notes

The Harmonic Duality of Symmetrical Sets

Symmetrical sets—especially those with a single line of symmetry—can, depending on their voicing, take on characteristics of both sharp-projecting and flat-projecting modalities. As a result, they are harmonically versatile and of particular interest to composers for their chameleon-like nature. Below are several examples of symmetrical sets in contrasting voicings (Figure 20-11 through Figure 20-14).

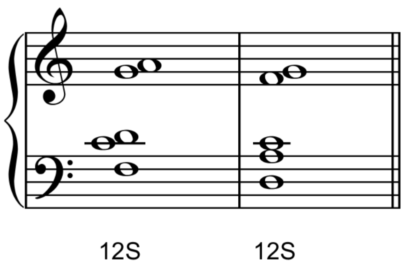


Figure 20-11. Pentatonic 12S (F 5-35)

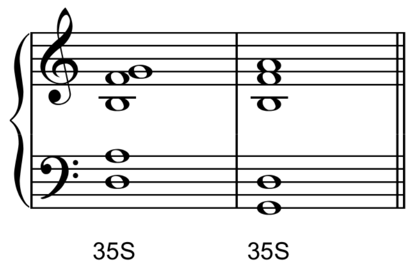


Figure 20-12. Dominant 9th chord, 35S (F 5-34)

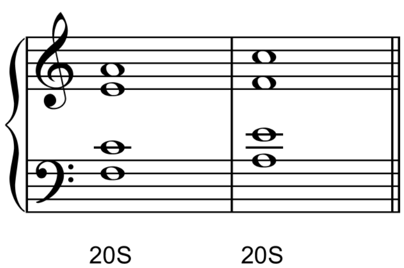


Figure 20-13. Major 7th chord, 20S (F 4-20)

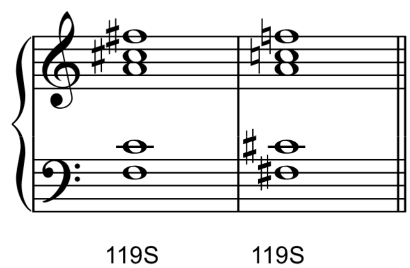


Figure 20-14. Pentachord 119S (F 5-22)

The most ubiquitous example of a symmetrical set is the diatonic set, 42S (F 7-35), also known as the major scale, the minor scale, or the various church modes (Figure 20-15).

Its sharp-projecting and flat-projecting qualities have been exploited by composers for centuries, and examples throughout the musical literature demonstrate the set's remarkable ability to shift fluidly between its two harmonic modalities (colloquially referred to as major and minor).

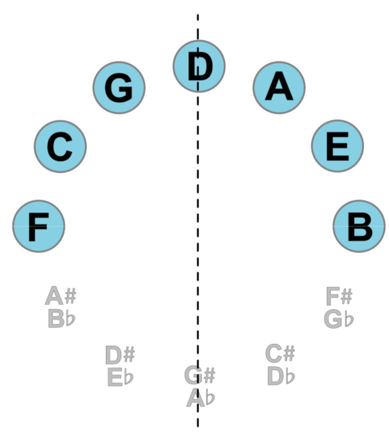
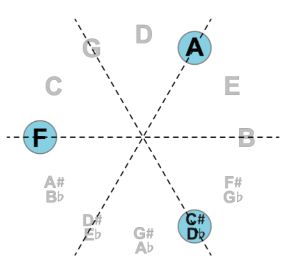
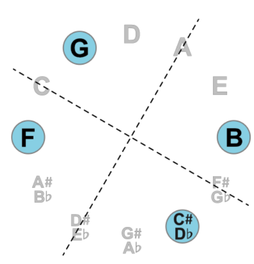


Figure 20-15. The symmetry of the diatonic set, 42S (F 7-35)

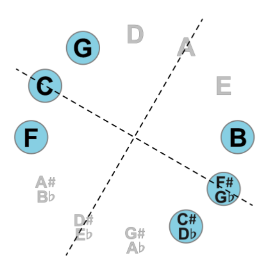
Harmonic Ambiguity of Sets with Multiple Lines of Symmetry



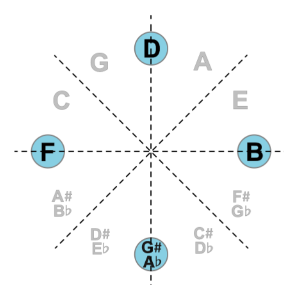
77S (F3-12)
Augmented Triad



97S (F 4-25)
French 6th chord



129S (F 6-7)
Messiaen's 5th mode



135S (F 4-28)
Diminished 7th chord

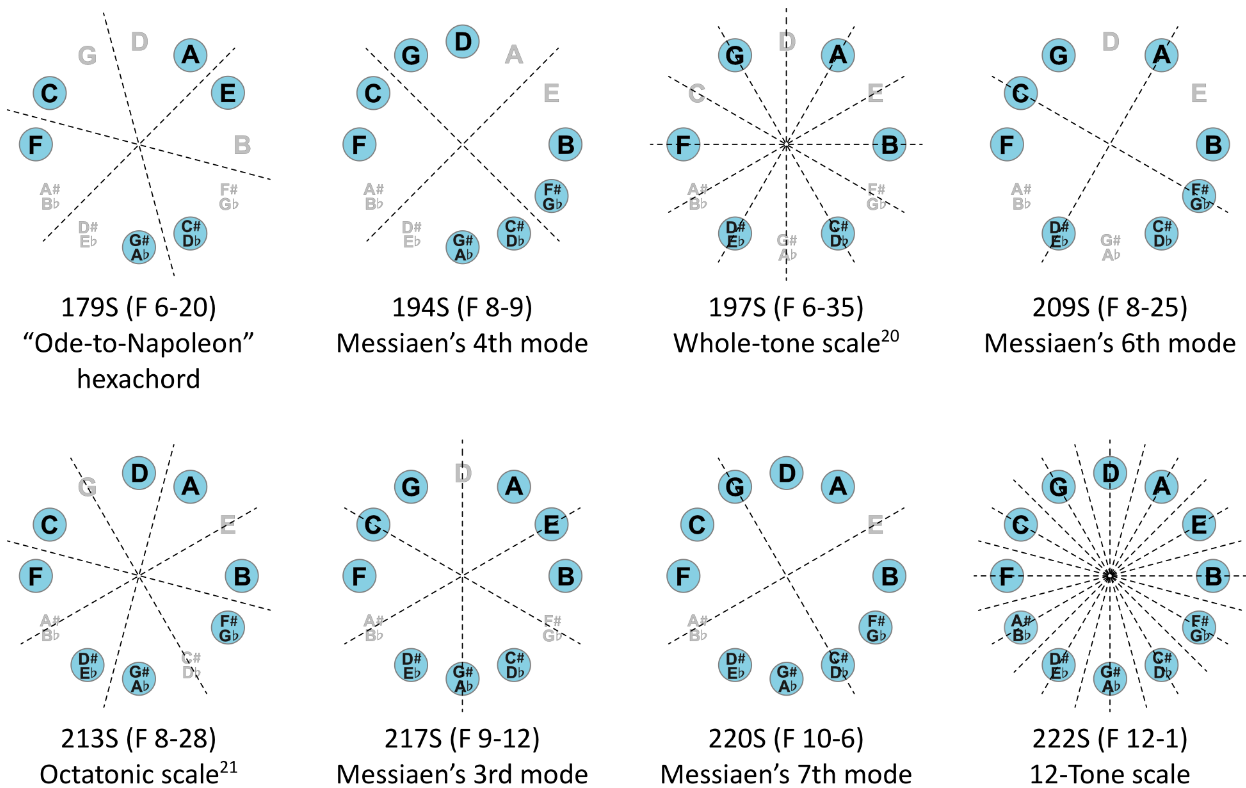


Figure 20-16. Symmetrical sets with multiple lines of symmetry

Symmetrical sets with multiple lines of symmetry are far more harmonically ambiguous and are therefore sought out by composers who wish to avoid allusions to diatonicism or tonal centers. Above is a list of all twelve symmetrical sets with multiple lines of symmetry, among them Olivier Messiaen’s modes of limited transposition (Figure 20-16).

In Practice

- Examine the following tables:
  - **Table 7** - The Natural Harmonic Procession of Symmetrical Sets in Sharp-Projecting Form
  - **Table 8** - The Natural Harmonic Procession of Symmetrical Sets in Flat-Projecting Form
- From the above-mentioned tables, choose three symmetrical sets and map them on the circle of fifths.
- Write out these sets in music notation, both as chords and as scales.
- Arrange the chords in a variety of voicings to highlight the different harmonic colors inherent in each set. Improvise or compose using the newly discovered sonorities.

[20] Messiaen’s 1st mode
[21] Messiaen’s 2nd mode

21. Mirror Sets

Characteristics of Mirror Sets

Every sharp-projecting set has a corresponding flat-projecting set; they function as each other's mirrors. Every symmetrical set, however, is its own mirror, since it can be classified as both a sharp-projecting and a flat-projecting set.

The simplest example of mirror sets is the pair formed by the sharp-projecting major triad, 8# (F 3-11B), and its counterpart, the flat-projecting minor triad, 8♭ (F 3-11A). In the table below, each is presented in its quintal prime form notated on the row of fifths: the sharp-projecting major triad beginning on F, and the flat-projecting minor triad beginning on B, with the mirror line crossing the D–G# axis (Figure 21-1 through Figure 21-3).

HP	Forte	Name	Numeric Quintal Prime Form				Mirror			
8#	3-11B	Major Triad	#1,2,5	F	C			A		
8♭	3-11A	Minor Triad	♭1,2,5			G			E	B

Figure 21-1. Major and minor triads, in quintal prime form

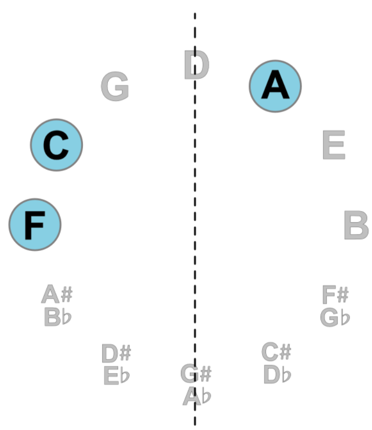


Figure 21-2. The sharp-projecting major triad, 8# (F 3-11B)

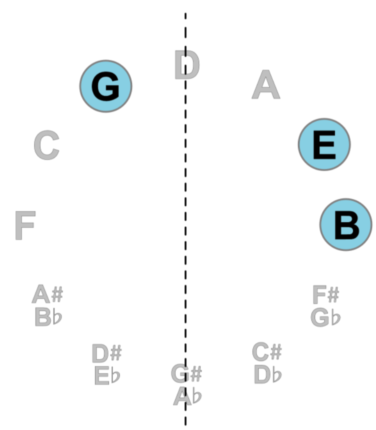


Figure 21-3. The flat-projecting minor triad, 8♭ (F 3-11A)

For mirror sets in their quintal prime form, the line of symmetry on the circle of fifths always passes through the vertical D–G# (north-south) axis. Their numeric quintal prime forms are represented by the same string of numbers, but with opposing #/♭ designations.

Below is another pair of mirror sets: the sharp-projecting pentachord 30# (F 5-29A), and its counterpart, the flat-projecting Insen scale, 30♭ (F 5-29B), (Figure 21-4 through Figure 21-6).

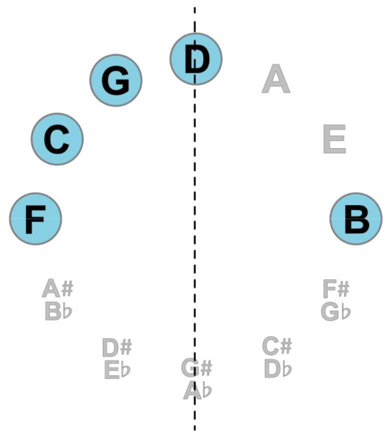


Figure 21-4. The sharp-projecting pentachord 30# (F 5-29A), in quintal prime form

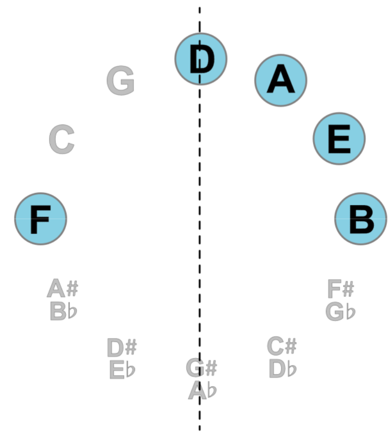


Figure 21-5. The flat-projecting Insen scale, 30♭ (F 5-29B), in quintal prime form

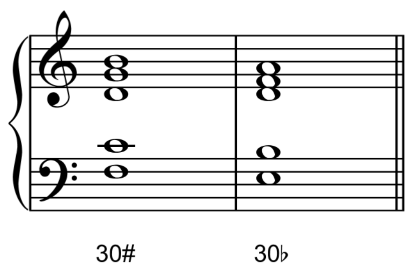


Figure 21-6. Pentachord 30# (F 5-29A), and Insen scale, 30♭ (F 5-29B), in quintal prime form

Mirror sets exhibit identical interval-class content and identical dissonance levels, yet they represent different modalities. Their classification is not merely an intellectual exercise but a practical tool for pairing closely related sets, offering composers a subtle shift in harmonic color. In this respect, mirror sets resemble symmetrical sets (harmonic duality), except that each mirror set requires its counterpart to effect the harmonic shift, whereas a symmetrical set can accomplish the shift on its own. In the Harmonic Processions tables, mirror sets appear frequently, as illustrated by the duality table in Chapter 25 and throughout the tables in the second half of this book. All mirror pairs share identical Harmonic-Processions numbers for ease of reference.

Let us highlight another pair of mirror sets, this time a pair of hexachords: the augmented 11th, 106# (F 6-34B), and its counterpart, the Prometheus chord, 106♭ (F 6-34A) (Figure 21-7 and Figure 21-8).

HP	Forte	Name	Numeric Quintal Prime Form						Mirror					
106#	6-34B	Augmented 11th (c)	#1,3,4,5,7,9			F		G	D	A		B		C#
106♭	6-34A	Prometheus (s)	♭1,3,4,5,7,9	E♭		F		G	D	A		B		

Figure 21-7. Augmented 11th hexachord, 106# (F 6-34B), and the Prometheus hexachord, 106♭ (F 6-34A)

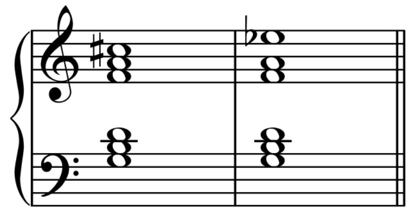


Figure 21-8. Augmented 11th hexachord, 106# (F 6-34B),
and the Prometheus hexachord, 106♭ (F 6-34A)

The Potential of Mirror Sets

Although the quintal prime forms of mirror sets always reflect across the D–G# axis, it is often useful to experiment with alternative axes. When a pronounced shift of modality is desired, the most effective strategy is to preserve the core as the common-tone anchor between the sets and to slide only the projecting notes across the chosen axis.

Consider again the pairing of the pentachord 30# (F 5-29A), with the Insen scale, 30♭ (F 5-29B), but this time using a different line of symmetry (Figure 21-9 through Figure 21-11). Notice that the resulting shift in modality is more pronounced than before.

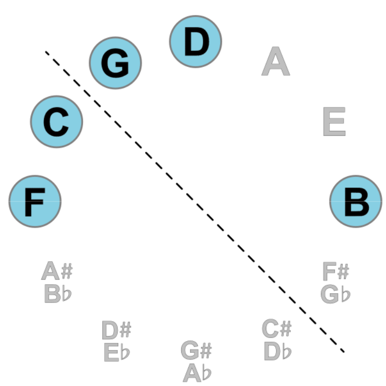


Figure 21-9. Pentachord 30# (F 5-29A),
with a new line of symmetry

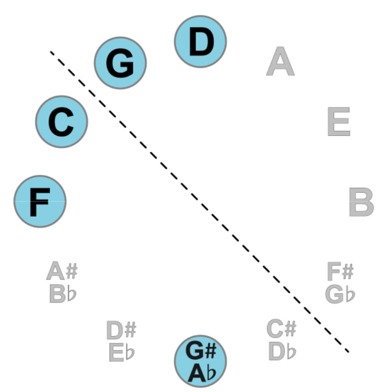


Figure 21-10. Insen scale, 30♭ (F 5-29B),
Transposed to align with the core
of pentachord 30#

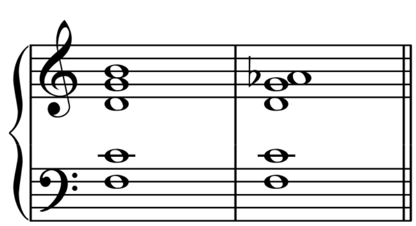
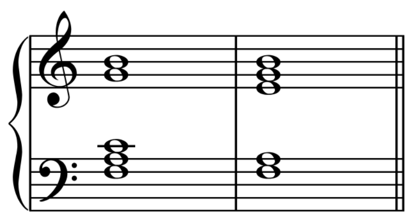


Figure 21-11. Pentachord 30# (F 5-29A), and Insen scale, 30♭ (F 5-29B), with aligned cores

Now let us compare another pair of mirror pentachords: the Lydian pentachord, 33# (F 5-24B), and its counterpart, the Phrygian pentachord, 33♭ (F 5-24A) (Figure 21-12 and Figure 21-13).

HP	Forte	Name	Numeric Quintal Prime Form				Mirror			
33#	5-24B	Lydian Pentachord	#1,2,3,5,7	F	C	G		A		B
33♭	5-24A	Phrygian Pentachord	♭1,2,3,5,7	F		G		A	E	B

Figure 21-12. Lydian pentachord, 33# (F 5-24B), and Phrygian pentachord, 33♭ (F 5-24A), in quintal prime form



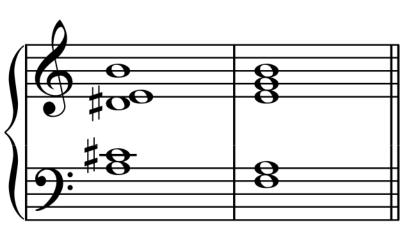
33# 33♭

Figure 21-13. Lydian pentachord, 33# (F 5-24B), and Phrygian pentachord, 33♭ (F 5-24A), in quintal prime form

When compared in their quintal prime form, the two pentachords exhibit only minimal harmonic distinction. But let us now align their cores by transposing the first pentachord so that the projections—and therefore the modalities—can be perceived more clearly (Figure 21-14 through Figure 21-16).

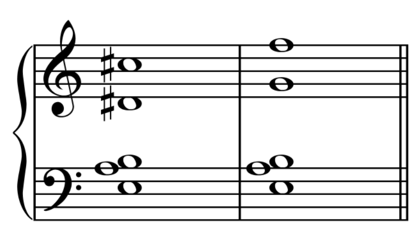
HP	Forte	Name	Numeric Quintal Prime Form						Mirror					
33#	5-24B	Lydian Pentachord	#1,2,3,5,7					A	E	B		C#		D#
33♭	5-24A	Phrygian Pentachord	♭1,2,3,5,7	F		G		A	E	B				

Figure 21-14. Lydian pentachord, 33# (F 5-24B), and Phrygian pentachord, 33♭ (F 5-24A), with aligned cores



33# 33♭

Figure 21-15. Lydian pentachord, 33# (F 5-24B), and Phrygian pentachord, 33♭ (F 5-24A), voicing 1



33# 33♭

Figure 21-16. Lydian pentachord, 33# (F 5-24B), and Phrygian pentachord, 33♭ (F 5-24A), voicing 2

In Practice

- Study **Table 1** - The Natural Harmonic Procession of All Sets in Dual Exposition.
- Select three pairs of non-symmetrical mirror sets from the table and write them in keyboard notation in their quintal prime form.
- Experiment with the selected sets using a variety of voicings and transpositions.

22. The Natural Harmonic Procession

Procession Defined

A procession is a deliberate and often solemn movement of individuals or groups advancing in a structured manner—typically as part of a ritual, celebration, or commemorative event. It embodies a sense of purpose, unity, and symbolism, transforming collective motion into a visual and emotional expression of identity, reverence, or transition.

In religious or spiritual contexts, processions mark sacred rites and evoke a sense of transcendence and devotion. In civic or cultural ceremonies, they honor historical events, revered figures, or shared communal values. Whatever the occasion, a procession follows a predetermined pathway and a deliberate sequence of participants—usually arranged according to hierarchy, role, or symbolic significance.

The Harmonic Processions are a system for classifying all 350 possible sets—commonly referred to as chords or scales—found in Western music. The name reflects the structural logic of the system, in which harmonies "process" sequentially from simple to complex, revealing shared origins, interrelationships, and distinct harmonic modalities. This approach offers a unified and comprehensive framework, treating all harmonies as a deliberate and interdependent whole. It uncovers the systemic elegance and purposeful architecture embedded in the very fabric of musical harmony.

The Methodology of the Natural Harmonic Procession

The Natural Harmonic Procession includes all 350 unique sets, beginning with the most consonant dyads and trichords and progressing toward the most dissonant dodecachord—also known as the twelve-tone or chromatic scale. The term *Natural* acknowledges the system's origin in the inherent logic of mathematics and distinguishes this Harmonic Procession from the other Processions based on the circle of fifths, which will be introduced in later chapters.

The order of the Natural Harmonic Procession is determined by the span of each set, expressed through the decimal system. Sets with smaller spans enter the Procession first, while sets with larger spans do not advance until all sets of smaller span have been exhausted. The order of entry is calculated by assigning each span an order of magnitude (Figure 22-1).

For example, sets with a span of 3—that is, sets encompassing three adjacent notes on the circle of fifths—enter the Procession at the 100 level. Sets with a span of 5 enter at the 10,000 level, sets with a span of 7 enter at the 1,000,000 level, and so on.

Span	Order of Magnitude
1	1
2	10
3	100
4	1,000
5	10,000
6	100,000
7	1,000,000
8	10,000,000
9	100,000,000
10	1,000,000,000
11	10,000,000,000
12	100,000,000,000

Figure 22-1. The orders of magnitude assigned to the spans of sets

At every order of magnitude, the binary system is employed: the presence of a note is represented by 1, and its absence by 0. In this way, each set receives an entry number in the Natural Harmonic Procession that reflects its harmonic complexity. The wider the span, the greater the complexity, and therefore the higher the number—and the later the entry—assigned to the set. These entry numbers express not only the overall span of the set but also encode the span of every individual note within it.

The system of the Natural Harmonic Procession thus provides an objective, mathematically grounded ordering of all sets based on close and distant quintal relationships as modeled on the circle of fifths. It reveals the unfolding of a fractal process[22], with the circle of fifths serving as its generative pathway.

In Practice

- Determine the spans of the following sets and calculate their level of entry using the appropriate order of magnitude:
  - F–C–G–A–E
  - F–C–D–A–B–F#–G#
  - B–E–G
  - B–A–G–F

[22] The fractal process is discussed in Chapter 26.

23. The Natural Order of Sharp-Projecting Sets

Calculating Entry Numbers into the Procession

As discussed in Chapter 22, the order of sets within the Natural Harmonic Procession is determined by assigning each note on the circle of fifths a successive order of magnitude. For sharp-projecting sets—those whose quintal prime form begins on F—this sequence always starts with F and proceeds clockwise around the circle of fifths. For example, sets with a span of 3 (spanning from F to G on the circle) enter the Procession at the 100 level; sets with a span of 5 (spanning from F to A) enter at the 10,000 level; and sets with a span of 7 (spanning from F to B) enter at the 1,000,000 level, and so on (Figure 23-1).

Span	Note	Order of Magnitude
1	F	1
2	C	10
3	G	100
4	D	1,000
5	A	10,000
6	E	100,000
7	B	1,000,000
8	**F#**	**10,000,000**
9	**C#**	**100,000,000**
10	**G#**	**1,000,000,000**
11	**D#**	**10,000,000,000**
12	**A#**	**100,000,000,000**

Figure 23-1. Orders of magnitude assigned to notes on the circle of fifths in the Natural Harmonic Procession of sharp-projecting sets

At every order of magnitude, the binary system is applied: 1 signifies the presence of a note, and 0 signifies its absence. The following example illustrates how the entry number for the set F–C–A—the major triad, 8# (F 3-11B)—is calculated (Figure 23-2):

Span	Note	Order of Magnitude	Note Present?	Integer
1	F	1	Yes	1
2	C	10	Yes	1
3	G	100	No	0
4	D	1,000	No	0
5	A	10,000	Yes	1

F + C + A = 1 + 10 + 10,000 = **10,011**

Figure 23-2. Calculation of the entry number of triad F–C–A in the Natural Harmonic Procession

The same can be calculated for the pentatonic set F–C–G–D–A, 12S (F 5-35), (Figure 23-3).

Span	Note	Order of Magnitude	Note Present?	Integer
1	F	1	Yes	1
2	C	10	Yes	1
3	G	100	Yes	1
4	D	1,000	Yes	1
5	A	10,000	Yes	1

F + C + G + D + A = 1 + 10 + 100 + 1,000 + 10,000 = **11,111**

Figure 23-3. Calculation of the entry number of the Pentatonic set F–C–G–D–A in the Natural Harmonic Procession

The pentatonic set F–C–G–D–A has a Natural Harmonic Procession entry number of 11,111 and therefore processes later than the major triad F–C–A, whose entry number is 10,011. Although both sets share the same span of 5, the pentatonic set is more complex because it contains more notes.

Below is a partial Natural Harmonic Procession of the sharp-projecting sets in quintal prime form. Both the major triad (F–C–A) and the pentatonic set (F–C–G–D–A) are marked to highlight their respective entries (Figure 23-4).

	Entry Number	1	10	100	1,000	10,000	100,000	1,000,000	10,000,000	100,000,000	1,000,000,000	10,000,000,000	100,000,000,000
	11	F	C										
	101	F		G									
	111	F	C	G									
	1,001	F			D								
	1,011	F	C		D								
	1,111	F	C	G	D								
	10,001	F				A							
Major Triad	10,011	F	C			A							
	10,101	F		G		A							
	10,111	F	C	G		A							
	11,011	F	C		D	A							
Pentatonic	11,111	F	C	G	D	A							
	100,001	F					E						
	100,011	F	C				E						
	100,101	F		G			E						
	100,111	F	C	G			E						
	101,011	F	C		D		E						
	101,101	F		G	D		E						
	101,111	F	C	G	D		E						

110,011	F	C			A	E						
110,111	F	C	G		A	E						
111,111	F	C	G	D	A	E						
1,000,001	F						B					
1,000,011	F	C					B					
1,000,101	F		G				B					
1,000,111	F	C	G				B					
1,001,001	F			D			B					
1,001,011	F	C		D			B					
1,001,101	F		G	D			B					
1,001,111	F	C	G	D			B					
1,010,011	F	C			A		B					
1,010,101	F		G		A		B					
1,010,111	F	C	G		A		B					
1,011,011	F	C		D	A		B					
1,011,101	F		G	D	A		B					
1,011,111	F	C	G	D	A		B					
1,100,011	F	C				E	B					
1,100,111	F	C	G			E	B					
1,101,011	F	C		D		E	B					
1,101,111	F	C	G	D		E	B					
1,110,111	F	C	G		A	E	B					
1,111,111	F	C	G	D	A	E	B					
10,000,101	F		G					F#				
10,000,111	F	C	G					F#				
10,001,001	F			D				F#				
10,001,011	F	C		D				F#				
10,001,101	F		G	D				F#				
10,001,111	F	C	G	D				F#				
10,010,011	F	C			A			F#				
10,010,101	F		G		A			F#				
10,010,111	F	C	G		A			F#				
10,011,001	F			D	A			F#				
10,011,011	F	C		D	A			F#				
10,011,101	F		G	D	A			F#				
10,011,111	F	C	G	D	A			F#				
10,100,011	F	C				E		F#				
10,100,101	F		G			E		F#				
10,100,111	F	C	G			E		F#				
10,101,011	F	C		D		E		F#				
10,101,101	F		G	D		E		F#				
10,101,111	F	C	G	D		E		F#				
10,110,011	F	C			A	E		F#				
10,110,111	F	C	G		A	E		F#				
10,111,011	F	C		D	A	E		F#				
etc.												

Figure 23-4. A partial Natural Harmonic Procession of the sharp-projecting sets in quintal prime form

Before we go any further, let us pause to admire the fractal pattern that emerges as the Natural Harmonic Procession unfolds. Below is a bird's-eye view of the Natural Harmonic Procession of the sharp-projecting sets (Figure 23-5).

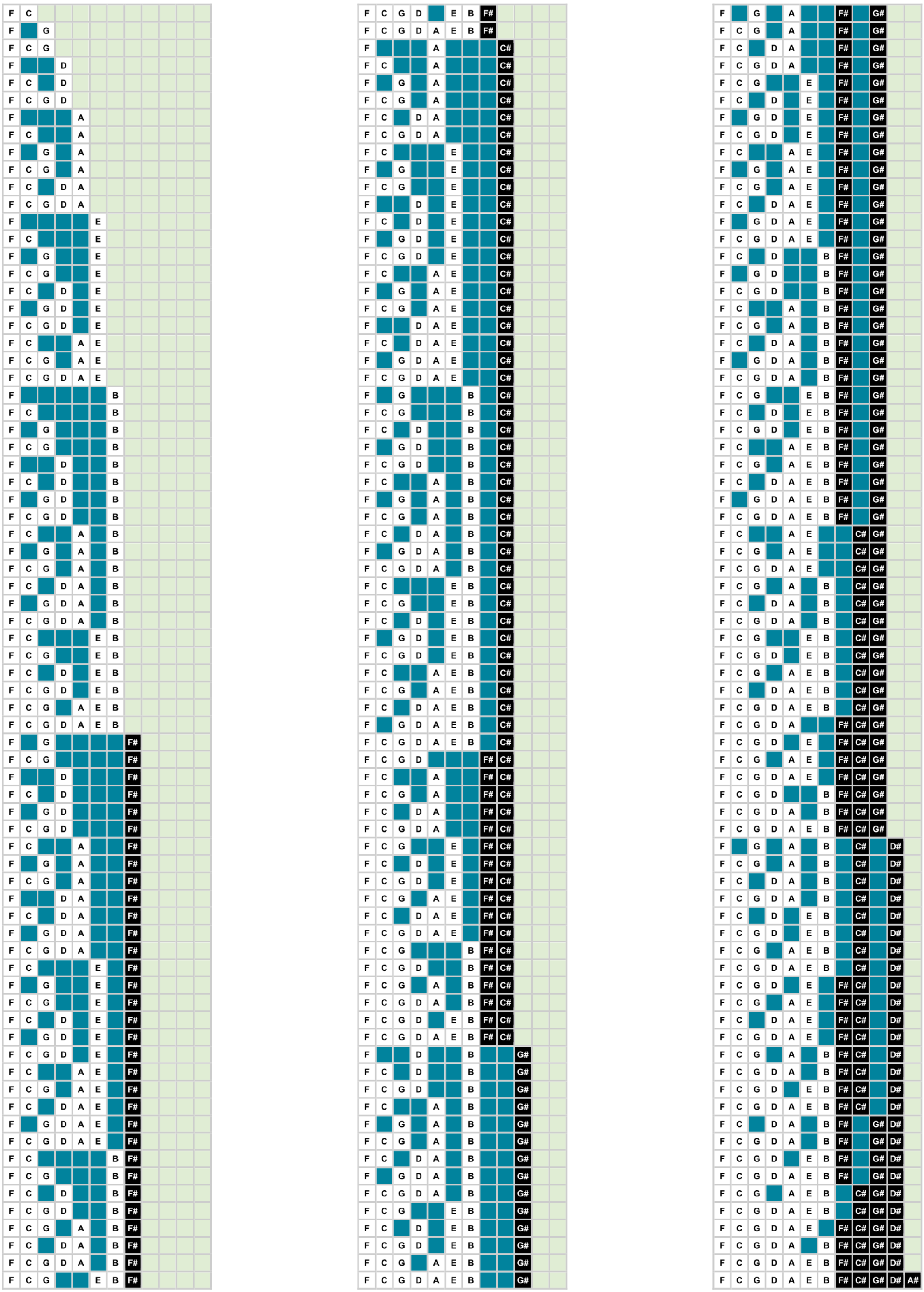


Figure 23-5. Bird's eye view of the Natural Harmonic Procession of the sharp-projecting sets

In Practice

- Calculate the entry numbers for the following sets and order the sets according to their complexity.
  - F–C–D–A–F#–C#
  - F–C–G–E
  - F–C–D–B–F#–G#
- Study **Table 5** - The Natural Harmonic Procession of Sharp-Projecting Sets.

24. The Natural Order of Flat-Projecting Sets

Calculating Entry Numbers into the Procession

The flat-projecting sets are ranked in the same manner as the sharp-projecting sets, except that their entry numbers are calculated beginning on B and proceeding counterclockwise around the circle of fifths. As before, each note on the circle corresponds to a successive order of magnitude, with 1 signifying the presence of a note and 0 signifying its absence (Figure 24-1).

Span	Note	Order of Magnitude
1	B	1
2	E	10
3	A	100
4	D	1,000
5	G	10,000
6	C	100,000
7	F	1,000,000
8	**B♭**	**10,000,000**
9	**E♭**	**100,000,000**
10	**A♭**	**1,000,000,000**
11	**D♭**	**10,000,000,000**
12	**G♭**	**100,000,000,000**

Figure 24-1. Orders of magnitude assigned to notes on the circle of fifths in the Natural Harmonic Procession of flat-projecting sets

Let us now calculate the entry number for the Alpha chord, 52♭S (F 4-17), B–D–G–B♭ (Figure 24-2):

Note	Order of Magnitude	Note Present?
B	1	1
E	10	0
A	100	0
D	1,000	1
G	10,000	1
C	100,000	0
F	1,000,000	0
B♭	**10,000,000**	**1**

B + D + G + B♭ = 1 + 1,000 + 10,000 + 10,000,000 = **10,011,001**

Figure 24-2. Calculation of the entry number of the Alpha chord 52♭S (F 4-17) in the Natural Harmonic Procession

Below is a partial Natural Harmonic Procession of the flat-projecting sets in quintal prime form. The entry of the Alpha chord is marked (Figure 24-3).

100,000,000,000	10,000,000,000	1,000,000,000	100,000,000	10,000,000	1,000,000	100,000	10,000	1,000	100	10	1	Entry Number
										E	B	11
									A		B	101
									A	E	B	111
								D			B	1,001
								D		E	B	1,011
								D	A	E	B	1,111
							G				B	10,001
							G			E	B	10,011
							G		A		B	10,101
							G		A	E	B	10,111
							G	D		E	B	11,011
							G	D	A	E	B	11,111
						C					B	100,001
						C				E	B	100,011
						C			A		B	100,101
						C			A	E	B	100,111
						C		D		E	B	101,011
						C		D	A		B	101,101
						C		D	A	E	B	101,111
						C	G			E	B	110,011
						C	G		A	E	B	110,111
						C	G	D	A	E	B	111,111
					F						B	1,000,001
					F					E	B	1,000,011
					F				A		B	1,000,101
					F				A	E	B	1,000,111
					F			D			B	1,001,001
					F			D		E	B	1,001,011
					F			D	A		B	1,001,101
					F			D	A	E	B	1,001,111
					F		G			E	B	1,010,011
					F		G		A		B	1,010,101
					F		G		A	E	B	1,010,111
					F		G	D		E	B	1,011,011
					F		G	D	A		B	1,011,101
					F		G	D	A	E	B	1,011,111
					F	C				E	B	1,100,011
					F	C			A	E	B	1,100,111
					F	C		D		E	B	1,101,011
					F	C		D	A	E	B	1,101,111
					F	C	G		A	E	B	1,110,111
					F	C	G	D	A	E	B	1,111,111
				B♭					A		B	10,000,101
				B♭					A	E	B	10,000,111
				B♭				D			B	10,001,001
				B♭				D		E	B	10,001,011

				B♭				D	A		B	10,001,101	
				B♭				D	A	E	B	10,001,111	
				B♭			G			E	B	10,010,011	
				B♭			G		A		B	10,010,101	
				B♭			G		A	E	B	10,010,111	
				B♭			G	D			B	10,011,001	**Alpha Chord**
				B♭			G	D		E	B	10,011,011	
				B♭			G	D	A		B	10,011,101	
				B♭			G	D	A	E	B	10,011,111	
				B♭		C				E	B	10,100,011	
				B♭		C			A		B	10,100,101	
				B♭		C			A	E	B	10,100,111	
				B♭		C		D		E	B	10,101,011	
				B♭		C		D	A		B	10,101,101	
				B♭		C		D	A	E	B	10,101,111	
				B♭		C	G			E	B	10,110,011	
				B♭		C	G		A	E	B	10,110,111	
				B♭		C	G	D		E	B	10,111,011	
				B♭		C	G	D	A		B	10,111,101	
				B♭		C	G	D	A	E	B	10,111,111	
												etc.	

Figure 24-3. A partial Natural Harmonic Procession of the flat-projecting sets in quintal prime form

The flat-projecting sets use the same sequence of entry numbers as the sharp-projecting sets. This is because, in the Natural Harmonic Procession, sets enter in simultaneous pairs: each sharp-projecting set is accompanied by its flat-projecting counterpart (its mirror). See Chapter 21 for an overview of mirror sets and Chapter 25 for a discussion of the duality within the Natural Harmonic Procession.

Harmonic-Processions Numbers

The larger the span of a set, the larger its entry number in the Procession. For sets with spans of 9, 10, 11, or 12, the entry numbers expand to nine, ten, eleven, and twelve digits respectively, and therefore become unwieldy when displayed in the tables. To conserve space and simplify presentation, once all entry numbers have been calculated and the order of all sets has been established, each set is then assigned a number between 1 and 222, together with a designation: # for sharp-projecting sets, ♭ for flat-projecting sets, and S for symmetrical sets. In the Harmonic Processions tables, only these smaller numbers (1-222)—referred to as the Harmonic-Processions numbers—are shown rather than the full entry numbers.

The Inventory of Sets

There are 350 unique sets, yet in the Harmonic Processions tables they are numbered 1-222. This is because the 256 unique non-symmetrical sets enter the Procession in pairs (mirror sets) and therefore share the same Harmonic-Processions number, distinguished only by the suffix # or ♭. By contrast, the 94 unique symmetrical sets—being their own mirrors—are listed twice, once in the sharp-projecting form and once in the flat-projecting form. Although they appear in both projections (for a total of 188 listings), they receive only 94 Harmonic-Processions numbers.

In total there are:

- 128 non-symmetrical pairs + 94 symmetrical sets (listed twice) = 222 pairs
- 256 unique non-symmetrical sets + 94 unique symmetrical sets = 350 unique sets

In Practice

- Study **Table 6** - The Natural Harmonic Procession of Flat-Projecting Sets.

- In **Table 2** - The Natural Harmonic Procession of All Sets, locate the following sharp-projecting sets and identify their flat-projecting mirrors:
  - Minor major 11th chord
  - German 6th chord
  - Locrian hexachord
  - Hungarian major scale

25. The Duality of the Natural Harmonic Procession

The Natural Harmonic Procession is a dual process in which two distinct movements can be observed: one formed by the sharp-projecting sets, which process from F in a clockwise direction, and the other formed by the flat-projecting sets, which process from B in a counterclockwise direction (Figure 25-1 and Figure 25-2).

Both the sharp-projecting and the flat-projecting sets advance simultaneously in paired motion. The two movements are symmetrical because the flat-projecting sets are mirror reflections of the sharp-projecting sets and therefore share the same levels of complexity and the same Harmonic-Processions numbers (1-222).

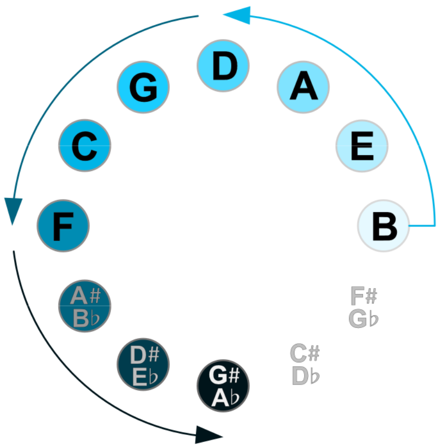


Figure 25-1. The pathway of the Natural Harmonic Procession of flat-projecting sets beginning with B

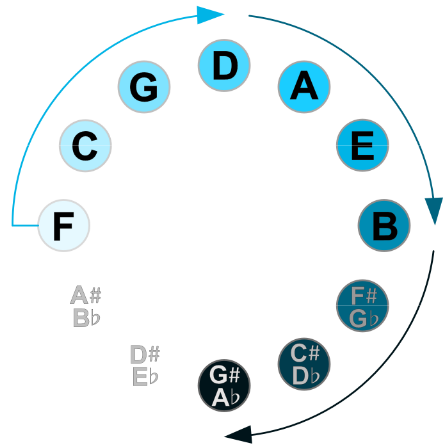


Figure 25-2. The pathway of the Natural Harmonic Procession of sharp-projecting sets beginning with F

Figure 25-3 illustrates the duality of the Natural Harmonic Procession.

Flat-Projecting Sets												HP	Sharp-Projecting Sets											
										E	B	1	F	C										
									A		B	2	F		G									
									A	E	B	3	F	C	G									
								D			B	4	F			D								
								D		E	B	5	F	C		D								
								D	A	E	B	6	F	C	G	D								
							G				B	7	F				A							
							G			E	B	8	F	C			A							
							G		A		B	9	F		G		A							
							G		A	E	B	10	F	C	G		A							
							G	D		E	B	11	F	C		D	A							
							G	D	A	E	B	12	F	C	G	D	A							
						C					B	13	F					E						
						C				E	B	14	F	C				E						
						C			A		B	15	F		G			E						

		C			A	E	B	16	F	C	G			E		
		C		D		E	B	17	F	C		D		E		
		C		D	A		B	18	F		G	D		E		
		C		D	A	E	B	19	F	C	G	D		E		
		C	G			E	B	20	F	C			A	E		
		C	G		A	E	B	21	F	C	G		A	E		
		C	G	D	A	E	B	22	F	C	G	D	A	E		
	F						B	23	F						B	
	F					E	B	24	F	C					B	
	F				A		B	25	F		G				B	
	F				A	E	B	26	F	C	G				B	
	F			D			B	27	F			D			B	
	F			D		E	B	28	F	C		D			B	
	F			D	A		B	29	F		G	D			B	
	F			D	A	E	B	30	F	C	G	D			B	
	F		G			E	B	31	F	C			A		B	
	F		G		A		B	32	F		G		A		B	
	F		G		A	E	B	33	F	C	G		A		B	
	F		G	D		E	B	34	F	C		D	A		B	
	F		G	D	A		B	35	F		G	D	A		B	
	F		G	D	A	E	B	36	F	C	G	D	A		B	
	F	C				E	B	37	F	C				E	B	
	F	C			A	E	B	38	F	C	G			E	B	
	F	C		D		E	B	39	F	C		D		E	B	
	F	C		D	A	E	B	40	F	C	G	D		E	B	
	F	C	G		A	E	B	41	F	C	G		A	E	B	
	F	C	G	D	A	E	B	42	F	C	G	D	A	E	B	
B♭					A		B	43	F		G					F#
B♭					A	E	B	44	F	C	G					F#
B♭				D			B	45	F			D				F#
B♭				D		E	B	46	F	C		D				F#
B♭				D	A		B	47	F		G	D				F#
B♭				D	A	E	B	48	F	C	G	D				F#
B♭			G			E	B	49	F	C			A			F#
B♭			G		A		B	50	F		G		A			F#
B♭			G		A	E	B	51	F	C	G		A			F#
B♭			G	D			B	52	F			D	A			F#
B♭			G	D		E	B	53	F	C		D	A			F#
B♭			G	D	A		B	54	F		G	D	A			F#
B♭			G	D	A	E	B	55	F	C	G	D	A			F#
B♭		C				E	B	56	F	C				E		F#
B♭		C			A		B	57	F		G			E		F#
B♭		C			A	E	B	58	F	C	G			E		F#
B♭		C		D		E	B	59	F	C		D		E		F#
B♭		C		D	A		B	60	F		G	D		E		F#
B♭		C		D	A	E	B	61	F	C	G	D		E		F#
B♭		C	G			E	B	62	F	C			A	E		F#
B♭		C	G		A	E	B	63	F	C	G		A	E		F#
B♭		C	G	D		E	B	64	F	C		D	A	E		F#
B♭		C	G	D	A		B	65	F		G	D	A	E		F#
B♭		C	G	D	A	E	B	66	F	C	G	D	A	E		F#
B♭	F					E	B	67	F	C					B	F#
B♭	F				A	E	B	68	F	C	G				B	F#
B♭	F			D		E	B	69	F	C		D			B	F#
B♭	F			D	A	E	B	70	F	C	G	D			B	F#
B♭	F		G		A	E	B	71	F	C	G		A		B	F#
B♭	F		G	D		E	B	72	F	C		D	A		B	F#
B♭	F		G	D	A	E	B	73	F	C	G	D	A		B	F#
B♭	F	C			A	E	B	74	F	C	G			E	B	F#

	B♭	F	C		D	A	E	B	75	F	C	G	D		E	B	F#	
	B♭	F	C	G	D	A	E	B	76	F	C	G	D	A	E	B	F#	
E♭				G				B	77	F				A				C#
E♭				G			E	B	78	F	C			A				C#
E♭				G		A		B	79	F		G		A				C#
E♭				G		A	E	B	80	F	C	G		A				C#
E♭				G	D		E	B	81	F	C		D	A				C#
E♭				G	D	A	E	B	82	F	C	G	D	A				C#
E♭			C				E	B	83	F	C				E			C#
E♭			C			A		B	84	F		G			E			C#
E♭			C			A	E	B	85	F	C	G			E			C#
E♭			C		D			B	86	F			D		E			C#
E♭			C		D		E	B	87	F	C		D		E			C#
E♭			C		D	A		B	88	F		G	D		E			C#
E♭			C		D	A	E	B	89	F	C	G	D		E			C#
E♭			C	G			E	B	90	F	C			A	E			C#
E♭			C	G		A		B	91	F		G		A	E			C#
E♭			C	G		A	E	B	92	F	C	G		A	E			C#
E♭			C	G	D			B	93	F			D	A	E			C#
E♭			C	G	D		E	B	94	F	C		D	A	E			C#
E♭			C	G	D	A		B	95	F		G	D	A	E			C#
E♭			C	G	D	A	E	B	96	F	C	G	D	A	E			C#
E♭		F				A		B	97	F		G				B		C#
E♭		F				A	E	B	98	F	C	G				B		C#
E♭		F			D		E	B	99	F	C		D			B		C#
E♭		F			D	A		B	100	F		G	D			B		C#
E♭		F			D	A	E	B	101	F	C	G	D			B		C#
E♭		F		G			E	B	102	F	C			A		B		C#
E♭		F		G		A		B	103	F		G		A		B		C#
E♭		F		G		A	E	B	104	F	C	G		A		B		C#
E♭		F		G	D		E	B	105	F	C		D	A		B		C#
E♭		F		G	D	A		B	106	F		G	D	A		B		C#
E♭		F		G	D	A	E	B	107	F	C	G	D	A		B		C#
E♭		F	C				E	B	108	F	C				E	B		C#
E♭		F	C			A	E	B	109	F	C	G			E	B		C#
E♭		F	C		D		E	B	110	F	C		D		E	B		C#
E♭		F	C		D	A		B	111	F		G	D		E	B		C#
E♭		F	C		D	A	E	B	112	F	C	G	D		E	B		C#
E♭		F	C	G			E	B	113	F	C			A	E	B		C#
E♭		F	C	G		A	E	B	114	F	C	G		A	E	B		C#
E♭		F	C	G	D		E	B	115	F	C		D	A	E	B		C#
E♭		F	C	G	D	A		B	116	F		G	D	A	E	B		C#
E♭		F	C	G	D	A	E	B	117	F	C	G	D	A	E	B		C#
E♭	B♭				D	A	E	B	118	F	C	G	D				F#	C#
E♭	B♭			G			E	B	119	F	C			A			F#	C#
E♭	B♭			G		A	E	B	120	F	C	G		A			F#	C#
E♭	B♭			G	D		E	B	121	F	C		D	A			F#	C#
E♭	B♭			G	D	A	E	B	122	F	C	G	D	A			F#	C#
E♭	B♭		C			A	E	B	123	F	C	G			E		F#	C#
E♭	B♭		C		D		E	B	124	F	C		D		E		F#	C#
E♭	B♭		C		D	A	E	B	125	F	C	G	D		E		F#	C#
E♭	B♭		C	G		A	E	B	126	F	C	G		A	E		F#	C#
E♭	B♭		C	G	D		E	B	127	F	C		D	A	E		F#	C#
E♭	B♭		C	G	D	A	E	B	128	F	C	G	D	A	E		F#	C#
E♭	B♭	F				A	E	B	129	F	C	G				B	F#	C#
E♭	B♭	F			D	A	E	B	130	F	C	G	D			B	F#	C#
E♭	B♭	F		G		A	E	B	131	F	C	G		A		B	F#	C#
E♭	B♭	F		G	D	A	E	B	132	F	C	G	D	A		B	F#	C#
E♭	B♭	F	C		D	A	E	B	133	F	C	G	D		E	B	F#	C#

	E♭	B♭	F	C	G	D	A	E	B	134	F	C	G	D	A	E	B	F#	C#	
A♭			F			D			B	135	F			D			B			G#
A♭			F			D		E	B	136	F	C		D			B			G#
A♭			F			D	A	E	B	137	F	C	G	D			B			G#
A♭			F		G			E	B	138	F	C			A		B			G#
A♭			F		G		A		B	139	F		G		A		B			G#
A♭			F		G		A	E	B	140	F	C	G		A		B			G#
A♭			F		G	D		E	B	141	F	C		D	A		B			G#
A♭			F		G	D	A		B	142	F		G	D	A		B			G#
A♭			F		G	D	A	E	B	143	F	C	G	D	A		B			G#
A♭			F	C			A	E	B	144	F	C	G			E	B			G#
A♭			F	C		D		E	B	145	F	C		D		E	B			G#
A♭			F	C		D	A	E	B	146	F	C	G	D		E	B			G#
A♭			F	C	G		A	E	B	147	F	C	G		A	E	B			G#
A♭			F	C	G	D	A	E	B	148	F	C	G	D	A	E	B			G#
A♭		B♭			G		A		B	149	F		G		A			F#		G#
A♭		B♭			G		A	E	B	150	F	C	G		A			F#		G#
A♭		B♭			G	D		E	B	151	F	C		D	A			F#		G#
A♭		B♭			G	D	A	E	B	152	F	C	G	D	A			F#		G#
A♭		B♭		C			A	E	B	153	F	C	G			E		F#		G#
A♭		B♭		C		D		E	B	154	F	C		D		E		F#		G#
A♭		B♭		C		D	A		B	155	F		G	D		E		F#		G#
A♭		B♭		C		D	A	E	B	156	F	C	G	D		E		F#		G#
A♭		B♭		C	G			E	B	157	F	C			A	E		F#		G#
A♭		B♭		C	G		A		B	158	F		G		A	E		F#		G#
A♭		B♭		C	G		A	E	B	159	F	C	G		A	E		F#		G#
A♭		B♭		C	G	D		E	B	160	F	C		D	A	E		F#		G#
A♭		B♭		C	G	D	A		B	161	F		G	D	A	E		F#		G#
A♭		B♭		C	G	D	A	E	B	162	F	C	G	D	A	E		F#		G#
A♭		B♭	F			D		E	B	163	F	C		D			B	F#		G#
A♭		B♭	F			D	A		B	164	F		G	D			B	F#		G#
A♭		B♭	F			D	A	E	B	165	F	C	G	D			B	F#		G#
A♭		B♭	F		G			E	B	166	F	C			A		B	F#		G#
A♭		B♭	F		G		A	E	B	167	F	C	G		A		B	F#		G#
A♭		B♭	F		G	D		E	B	168	F	C		D	A		B	F#		G#
A♭		B♭	F		G	D	A		B	169	F		G	D	A		B	F#		G#
A♭		B♭	F		G	D	A	E	B	170	F	C	G	D	A		B	F#		G#
A♭		B♭	F	C			A	E	B	171	F	C	G			E	B	F#		G#
A♭		B♭	F	C		D		E	B	172	F	C		D		E	B	F#		G#
A♭		B♭	F	C		D	A	E	B	173	F	C	G	D		E	B	F#		G#
A♭		B♭	F	C	G			E	B	174	F	C			A	E	B	F#		G#
A♭		B♭	F	C	G		A	E	B	175	F	C	G		A	E	B	F#		G#
A♭		B♭	F	C	G	D		E	B	176	F	C		D	A	E	B	F#		G#
A♭		B♭	F	C	G	D	A		B	177	F		G	D	A	E	B	F#		G#
A♭		B♭	F	C	G	D	A	E	B	178	F	C	G	D	A	E	B	F#		G#
A♭	E♭			C	G			E	B	179	F	C			A	E			C#	G#
A♭	E♭			C	G		A	E	B	180	F	C	G		A	E			C#	G#
A♭	E♭			C	G	D	A	E	B	181	F	C	G	D	A	E			C#	G#
A♭	E♭		F		G		A	E	B	182	F	C	G		A		B		C#	G#
A♭	E♭		F		G	D		E	B	183	F	C		D	A		B		C#	G#
A♭	E♭		F		G	D	A	E	B	184	F	C	G	D	A		B		C#	G#
A♭	E♭		F	C			A	E	B	185	F	C	G			E	B		C#	G#
A♭	E♭		F	C		D	A	E	B	186	F	C	G	D		E	B		C#	G#
A♭	E♭		F	C	G		A	E	B	187	F	C	G		A	E	B		C#	G#
A♭	E♭		F	C	G	D		E	B	188	F	C		D	A	E	B		C#	G#
A♭	E♭		F	C	G	D	A	E	B	189	F	C	G	D	A	E	B		C#	G#
A♭	E♭	B♭			G	D	A	E	B	190	F	C	G	D	A			F#	C#	G#
A♭	E♭	B♭		C		D	A	E	B	191	F	C	G	D		E		F#	C#	G#
A♭	E♭	B♭		C	G		A	E	B	192	F	C	G		A	E		F#	C#	G#

		A♭	E♭	B♭		C	G	D	A	E	B	193	F	C	G	D	A	E		F#	C#	G#		
		A♭	E♭	B♭	F			D	A	E	B	194	F	C	G	D			B	F#	C#	G#		
		A♭	E♭	B♭	F		G	D	A	E	B	195	F	C	G	D	A		B	F#	C#	G#		
		A♭	E♭	B♭	F	C	G	D	A	E	B	196	F	C	G	D	A	E	B	F#	C#	G#		
	D♭		E♭		F		G		A		B	197	F		G		A		B		C#		D#	
	D♭		E♭		F		G		A	E	B	198	F	C	G		A		B		C#		D#	
	D♭		E♭		F		G	D		E	B	199	F	C		D	A		B		C#		D#	
	D♭		E♭		F		G	D	A	E	B	200	F	C	G	D	A		B		C#		D#	
	D♭		E♭		F	C		D		E	B	201	F	C		D		E	B		C#		D#	
	D♭		E♭		F	C		D	A	E	B	202	F	C	G	D		E	B		C#		D#	
	D♭		E♭		F	C	G		A	E	B	203	F	C	G		A	E	B		C#		D#	
	D♭		E♭		F	C	G	D	A	E	B	204	F	C	G	D	A	E	B		C#		D#	
	D♭		E♭	B♭		C		D	A	E	B	205	F	C	G	D		E		F#	C#		D#	
	D♭		E♭	B♭		C	G		A	E	B	206	F	C	G		A	E		F#	C#		D#	
	D♭		E♭	B♭		C	G	D		E	B	207	F	C		D	A	E		F#	C#		D#	
	D♭		E♭	B♭		C	G	D	A	E	B	208	F	C	G	D	A	E		F#	C#		D#	
	D♭		E♭	B♭	F		G		A	E	B	209	F	C	G		A		B	F#	C#		D#	
	D♭		E♭	B♭	F		G	D	A	E	B	210	F	C	G	D	A		B	F#	C#		D#	
	D♭		E♭	B♭	F	C		D	A	E	B	211	F	C	G	D		E	B	F#	C#		D#	
	D♭		E♭	B♭	F	C	G	D	A	E	B	212	F	C	G	D	A	E	B	F#	C#		D#	
	D♭	A♭		B♭	F		G	D		E	B	213	F	C		D	A		B	F#		G#	D#	
	D♭	A♭		B♭	F		G	D	A	E	B	214	F	C	G	D	A		B	F#		G#	D#	
	D♭	A♭		B♭	F	C		D	A	E	B	215	F	C	G	D		E	B	F#		G#	D#	
	D♭	A♭		B♭	F	C	G	D	A	E	B	216	F	C	G	D	A	E	B	F#		G#	D#	
	D♭	A♭	E♭		F	C	G		A	E	B	217	F	C	G		A	E	B		C#	G#	D#	
	D♭	A♭	E♭		F	C	G	D	A	E	B	218	F	C	G	D	A	E	B		C#	G#	D#	
	D♭	A♭	E♭	B♭		C	G	D	A	E	B	219	F	C	G	D	A	E		F#	C#	G#	D#	
	D♭	A♭	E♭	B♭	F		G	D	A	E	B	220	F	C	G	D	A		B	F#	C#	G#	D#	
	D♭	A♭	E♭	B♭	F	C	G	D	A	E	B	221	F	C	G	D	A	E	B	F#	C#	G#	D#	
G♭	D♭	A♭	E♭	B♭	F	C	G	D	A	E	B	222	F	C	G	D	A	E	B	F#	C#	G#	D#	A#

Figure 25-3. The duality of the Natural Harmonic Procession table

In practice, because of the limitations of both paper and screen formats, the Harmonic Processions tables in the second half of the book present a collapsed view of the duality: the sharp-projecting and flat-projecting listings are interwoven. This consolidation allows sufficient space to include the associated names, modalities, and other characteristics of all sets.

In Practice

- In **Table 1** - The Natural Harmonic Procession of All Sets in Dual Exposition, locate the following sets:
  - Ionian hexachord
  - All-Interval tetrachord
  - Diatonic scale
  - Octatonic scale
  - Augmented triad
  - Whole-tone scale
  - Chromatic scale

- Study the Natural Harmonic Processions **Tables 9-19**.

26. Modalities and the Modal Harmonic Procession

The Fractal Process

As the Natural Harmonic Procession unfolds, discrete modalities emerge in a fractal process encoded in the structure of deeply recursive binary counting. That recursion generates fractal geometry resembling the Sierpiński triangle—one of the most well-known discrete fractals—and produces self-similar triangular voids. The same underlying structure appears in Pascal's triangle mod 2, the Rule 90 cellular automaton, XOR-based patterns, and many other recursive combinatorial objects.

Let us look at the Ionian Hexachord # modality below (Figure 26-1), and observe how the fractal process unfolds. The fractal process is initiated by the span interval—the distance between the core note F and its furthest projection, E, the harmonic boundary of the modality. The process then continues with the successive additions of perfect fifths (C, G, D, and A) until all note-slots are filled and the recursion terminates—more precisely, in the language of fractals, until the finite truncation of the recursive process has occurred. At that point, the fractal has reached its limit for this particular span in the set F–C–G–D–A–E, known as the Ionian Hexachord, 22#S (F 6-32).

HP	Forte	Name	Modality	Numeric Quintal Prime Form	Quintal Prime Form					
13#S	2-1	Minor Second (m2)	Ionian Hexachord #	#1,6	F					E
14#	3-4B		Ionian Hexachord #	#1,2,6	F	C				E
15#	3-2A		Ionian Hexachord #	#1,3,6	F		G			E
16#	4-14B		Ionian Hexachord #	#1,2,3,6	F	C	G			E
17#	4-11B	Ionian Tetrachord	Ionian Hexachord #	#1,2,4,6	F	C		D		E
18#S	4-10	Aeolian Tetrachord	Ionian Hexachord #	#1,3,4,6	F		G	D		E
19#	5-23B	Ionian Pentachord	Ionian Hexachord #	#1,2,3,4,6	F	C	G	D		E
20#S	4-20	Major 7th (c)	Ionian Hexachord #	#1,2,5,6	F	C			A	E
21#	5-27A	Major 9th (c)	Ionian Hexachord #	#1,2,3,5,6	F	C	G		A	E
22#S	6-32	Ionian Hexachord	Ionian Hexachord #	#1,2,3,4,5,6	F	C	G	D	A	E

Figure 26-1. The fractal pattern of the Ionian Hexachord # modality

Now let us turn to the Diatonic # modality (Figure 26-2). This modality begins with the span interval F–B, its harmonic boundary. From there, we observe the succession of perfect fifths radiating outward from the core note F toward the furthest projection, B. The modality unfolds in a fractal pattern until it reaches its limit—when all note-slots are occupied—in the diatonic set F–C–G–D–A–E–B, 42#S (F 7-35).

There are multiple modalities within the Natural Harmonic Procession of the sharp-projecting sets. Some are quite limited—such as the Suspended Triad # modality—while others are vast and intricate, such as the Enigmatic modality, which contains multiple sub-modalities. The wider the initial span of a modality, the greater its potential for complexity, and the more room it provides for the recursive fractal patterns inherited from preceding modalities.

Although each modality expresses a distinct harmonic flavor, they are constructed in a manner reminiscent of the Russian nesting dolls: one modality is embedded within another, and another within that, and so on. For example, the Diatonic # modality contains characteristics of the Ionian Hexachord # modality that precedes it; the Lydian modality contains characteristics of both the Ionian Hexachord # and the Diatonic # modalities; and the Enigmatic modality contains features of all the preceding ones. At the later stages of the Natural Harmonic Procession, the fractal process becomes so complex that it is helpful to divide the larger spans (such as F–G# and F–D#) into separate modalities based on the chromatic content of their projections.

HP	Forte	Name	Modality	Numeric Quintal Prime Form	Quintal Prime Form						
23#S	2-6	Tritone (TT)	Diatonic #	#1,7	F						B
24#	3-5A	Viennese Trichord	Diatonic #	#1,2,7	F	C					B
25#	3-8A	Italian 6th (c)	Diatonic #	#1,3,7	F		G				B
26#	4-16B		Diatonic #	#1,2,3,7	F	C	G				B
27#S	3-10	Diminished Triad	Diatonic #	#1,4,7	F			D			B
28#	4-13A		Diatonic #	#1,2,4,7	F	C		D			B
29#	4-27B	German 6th (c)	Diatonic #	#1,3,4,7	F		G	D			B
30#	5-29A		Diatonic #	#1,2,3,4,7	F	C	G	D			B
31#	4-Z29B	All-Interval Tetrachord	Diatonic #	#1,2,5,7	F	C			A		B
32#S	4-21	Whole-Tone Tetrachord	Diatonic #	#1,3,5,7	F		G		A		B
33#	5-24B	Lydian Pentachord	Diatonic #	#1,2,3,5,7	F	C	G		A		B
34#	5-25A	Half-Diminished Minor 9th (c)	Diatonic #	#1,2,4,5,7	F	C		D	A		B
35#S	5-34	Dominant 9th (c)	Diatonic #	#1,3,4,5,7	F		G	D	A		B
36#	6-33B	Lydian Hexachord	Diatonic #	#1,2,3,4,5,7	F	C	G	D	A		B
37#S	4-8		Diatonic #	#1,2,6,7	F	C				E	B
38#	5-20A		Diatonic #	#1,2,3,6,7	F	C	G			E	B
39#S	5-Z12	Locrian Pentachord	Diatonic #	#1,2,4,6,7	F	C		D		E	B
40#	6-Z25A	Locrian Hexachord	Diatonic #	#1,2,3,4,6,7	F	C	G	D		E	B
41#S	6-Z26	Phrygian Hexachord	Diatonic #	#1,2,3,5,6,7	F	C	G		A	E	B
42#S	7-35	Diatonic (s)	Diatonic #	#1,2,3,4,5,6,7	F	C	G	D	A	E	B

Figure 26-2. The fractal pattern of the Diatonic # modality

Classification of Modalities

The classifications of modalities are not merely theoretical. They organize sonorities into related groups, each carrying a distinct harmonic flavor. Within the Natural Harmonic Procession, we can distinguish 36 such modalities. They are presented in tables below (Figure 26-3 through Figure 26-5).

No.	Modality ♭	Quintal Prime Form								HP	Quintal Prime Form								Modality #	No.
	--							E	B	1	F	C							--	
2	Sus. Triad ♭[23]						A		B	2	F		G						Sus. Triad #[24]	1
							A	E	B	3	F	C	G							
4	Quartal ♭					D			B	4	F			D					Quartal #	3
						D		E	B	5	F	C		D						
						D	A	E	B	6	F	C	G	D						
6	Pentatonic ♭				G				B	7	F				A				Pentatonic #	5
					G			E	B	8	F	C			A					
					G		A		B	9	F		G		A					
					G		A	E	B	10	F	C	G		A					
					G	D		E	B	11	F	C		D	A					
					G	D	A	E	B	12	F	C	G	D	A					
8	Ionian Hexachord ♭			C					B	13	F					E			Ionian Hexachord #	7
				C				E	B	14	F	C				E				
				C			A		B	15	F		G			E				
				C			A	E	B	16	F	C	G			E				
				C		D		E	B	17	F	C		D		E				
				C		D	A		B	18	F		G	D		E				
				C		D	A	E	B	19	F	C	G	D		E				
				C	G			E	B	20	F	C			A	E				
				C	G		A	E	B	21	F	C	G		A	E				
				C	G	D	A	E	B	22	F	C	G	D	A	E				
10	Diatonic ♭		F						B	23	F						B		Diatonic #	9
			F					E	B	24	F	C					B			
			F				A		B	25	F		G				B			
			F				A	E	B	26	F	C	G				B			
			F			D			B	27	F			D			B			
			F			D		E	B	28	F	C		D			B			
			F			D	A		B	29	F		G	D			B			
			F			D	A	E	B	30	F	C	G	D			B			
			F		G			E	B	31	F	C			A		B			
			F		G		A		B	32	F		G		A		B			
			F		G		A	E	B	33	F	C	G		A		B			
			F		G	D		E	B	34	F	C		D	A		B			
			F		G	D	A		B	35	F		G	D	A		B			
			F		G	D	A	E	B	36	F	C	G	D	A		B			
			F	C				E	B	37	F	C				E	B			
			F	C			A	E	B	38	F	C	G			E	B			
			F	C		D		E	B	39	F	C		D		E	B			
			F	C		D	A	E	B	40	F	C	G	D		E	B			
			F	C	G		A	E	B	41	F	C	G		A	E	B			
			F	C	G	D	A	E	B	42	F	C	G	D	A	E	B			
12	Mixolydian	B♭					A		B	43	F		G					F#	Lydian	11
		B♭					A	E	B	44	F	C	G					F#		
		B♭				D			B	45	F			D				F#		
		B♭				D		E	B	46	F	C		D				F#		
		B♭				D	A		B	47	F		G	D				F#		
		B♭				D	A	E	B	48	F	C	G	D				F#		
		B♭			G			E	B	49	F	C			A			F#		
		B♭			G		A		B	50	F		G		A			F#		
		B♭			G		A	E	B	51	F	C	G		A			F#		

23 Suspended Triad ♭
24 Suspended Triad #

			B♭			G	D			B	52	F			D	A			F#			
			B♭			G	D		E	B	53	F	C		D	A			F#			
			B♭			G	D	A		B	54	F		G	D	A			F#			
			B♭			G	D	A	E	B	55	F	C	G	D	A			F#			
			B♭		C				E	B	56	F	C				E		F#			
			B♭		C			A		B	57	F		G			E		F#			
			B♭		C			A	E	B	58	F	C	G			E		F#			
			B♭		C		D		E	B	59	F	C		D		E		F#			
			B♭		C		D	A		B	60	F		G	D		E		F#			
			B♭		C		D	A	E	B	61	F	C	G	D		E		F#			
			B♭		C	G			E	B	62	F	C			A	E		F#			
			B♭		C	G		A	E	B	63	F	C	G		A	E		F#			
			B♭		C	G	D		E	B	64	F	C		D	A	E		F#			
			B♭		C	G	D	A		B	65	F		G	D	A	E		F#			
			B♭		C	G	D	A	E	B	66	F	C	G	D	A	E		F#			
			B♭	F					E	B	67	F	C					B	F#			
			B♭	F				A	E	B	68	F	C	G				B	F#			
			B♭	F			D		E	B	69	F	C		D			B	F#			
			B♭	F			D	A	E	B	70	F	C	G	D			B	F#			
			B♭	F		G		A	E	B	71	F	C	G		A		B	F#			
			B♭	F		G	D		E	B	72	F	C		D	A		B	F#			
			B♭	F		G	D	A	E	B	73	F	C	G	D	A		B	F#			
			B♭	F	C			A	E	B	74	F	C	G			E	B	F#			
			B♭	F	C		D	A	E	B	75	F	C	G	D		E	B	F#			
			B♭	F	C	G	D	A	E	B	76	F	C	G	D	A	E	B	F#			
14	Mystic	E♭				G				B	77	F				A				C#	Enigmatic	13
		E♭				G			E	B	78	F	C			A				C#		
		E♭				G		A		B	79	F		G		A				C#		
		E♭				G		A	E	B	80	F	C	G		A				C#		
		E♭				G	D		E	B	81	F	C		D	A				C#		
		E♭				G	D	A	E	B	82	F	C	G	D	A				C#		
		E♭			C				E	B	83	F	C				E			C#		
		E♭			C			A		B	84	F		G			E			C#		
		E♭			C			A	E	B	85	F	C	G			E			C#		
		E♭			C		D			B	86	F			D		E			C#		
		E♭			C		D		E	B	87	F	C		D		E			C#		
		E♭			C		D	A		B	88	F		G	D		E			C#		
		E♭			C		D	A	E	B	89	F	C	G	D		E			C#		
		E♭			C	G			E	B	90	F	C			A	E			C#		
		E♭			C	G		A		B	91	F		G		A	E			C#		
		E♭			C	G		A	E	B	92	F	C	G		A	E			C#		
		E♭			C	G	D			B	93	F			D	A	E			C#		
		E♭			C	G	D		E	B	94	F	C		D	A	E			C#		
		E♭			C	G	D	A		B	95	F		G	D	A	E			C#		
		E♭			C	G	D	A	E	B	96	F	C	G	D	A	E			C#		
		E♭		F				A		B	97	F		G				B		C#		
		E♭		F				A	E	B	98	F	C	G				B		C#		
		E♭		F			D		E	B	99	F	C		D			B		C#		
		E♭		F			D	A		B	100	F		G	D			B		C#		
		E♭		F			D	A	E	B	101	F	C	G	D			B		C#		
		E♭		F		G			E	B	102	F	C			A		B		C#		
		E♭		F		G		A		B	103	F		G		A		B		C#		
		E♭		F		G		A	E	B	104	F	C	G		A		B		C#		

			E♭		F		G	D		E	B	105	F	C		D	A		B		C#			
			E♭		F		G	D	A		B	106	F		G	D	A		B		C#			
			E♭		F		G	D	A	E	B	107	F	C	G	D	A		B		C#			
			E♭		F	C				E	B	108	F	C				E	B		C#			
			E♭		F	C			A	E	B	109	F	C	G			E	B		C#			
			E♭		F	C		D		E	B	110	F	C		D		E	B		C#			
			E♭		F	C		D	A		B	111	F		G	D		E	B		C#			
			E♭		F	C		D	A	E	B	112	F	C	G	D		E	B		C#			
			E♭		F	C	G			E	B	113	F	C			A	E	B		C#			
			E♭		F	C	G		A	E	B	114	F	C	G		A	E	B		C#			
			E♭		F	C	G	D		E	B	115	F	C		D	A	E	B		C#			
			E♭		F	C	G	D	A		B	116	F		G	D	A	E	B		C#			
			E♭		F	C	G	D	A	E	B	117	F	C	G	D	A	E	B		C#			
16	Blues ♭		E♭	B♭				D	A	E	B	118	F	C	G	D				F#	C#		Blues #	15
			E♭	B♭			G			E	B	119	F	C			A			F#	C#			
			E♭	B♭			G		A	E	B	120	F	C	G		A			F#	C#			
			E♭	B♭			G	D		E	B	121	F	C		D	A			F#	C#			
			E♭	B♭			G	D	A	E	B	122	F	C	G	D	A			F#	C#			
			E♭	B♭		C			A	E	B	123	F	C	G			E		F#	C#			
			E♭	B♭		C		D		E	B	124	F	C		D		E		F#	C#			
			E♭	B♭		C		D	A	E	B	125	F	C	G	D		E		F#	C#			
			E♭	B♭		C	G		A	E	B	126	F	C	G		A	E		F#	C#			
			E♭	B♭		C	G	D		E	B	127	F	C		D	A	E		F#	C#			
			E♭	B♭		C	G	D	A	E	B	128	F	C	G	D	A	E		F#	C#			
			E♭	B♭	F				A	E	B	129	F	C	G				B	F#	C#			
			E♭	B♭	F			D	A	E	B	130	F	C	G	D			B	F#	C#			
			E♭	B♭	F		G		A	E	B	131	F	C	G		A		B	F#	C#			
			E♭	B♭	F		G	D	A	E	B	132	F	C	G	D	A		B	F#	C#			
			E♭	B♭	F	C		D	A	E	B	133	F	C	G	D		E	B	F#	C#			
			E♭	B♭	F	C	G	D	A	E	B	134	F	C	G	D	A	E	B	F#	C#			
18	Diminished ♭	A♭			F			D			B	135	F			D			B			G#	Diminished #	17
		A♭			F			D		E	B	136	F	C		D			B			G#		
		A♭			F			D	A	E	B	137	F	C	G	D			B			G#		
		A♭			F		G			E	B	138	F	C			A		B			G#		
		A♭			F		G		A		B	139	F		G		A		B			G#		
		A♭			F		G		A	E	B	140	F	C	G		A		B			G#		
		A♭			F		G	D		E	B	141	F	C		D	A		B			G#		
		A♭			F		G	D	A		B	142	F		G	D	A		B			G#		
		A♭			F		G	D	A	E	B	143	F	C	G	D	A		B			G#		
		A♭			F	C			A	E	B	144	F	C	G			E	B			G#		
		A♭			F	C		D		E	B	145	F	C		D		E	B			G#		
		A♭			F	C		D	A	E	B	146	F	C	G	D		E	B			G#		
		A♭			F	C	G		A	E	B	147	F	C	G		A	E	B			G#		
		A♭			F	C	G	D	A	E	B	148	F	C	G	D	A	E	B			G#		
20	Romanian	A♭		B♭			G		A		B	149	F		G		A			F#		G#	Hungarian	19
		A♭		B♭			G		A	E	B	150	F	C	G		A			F#		G#		
		A♭		B♭			G	D		E	B	151	F	C		D	A			F#		G#		
		A♭		B♭			G	D	A	E	B	152	F	C	G	D	A			F#		G#		
		A♭		B♭		C			A	E	B	153	F	C	G			E		F#		G#		
		A♭		B♭		C		D		E	B	154	F	C		D		E		F#		G#		
		A♭		B♭		C		D	A		B	155	F		G	D		E		F#		G#		
		A♭		B♭		C		D	A	E	B	156	F	C	G	D		E		F#		G#		
		A♭		B♭		C	G			E	B	157	F	C			A	E		F#		G#		
		A♭		B♭		C	G		A		B	158	F		G		A	E		F#		G#		
		A♭		B♭		C	G		A	E	B	159	F	C	G		A	E		F#		G#		
		A♭		B♭		C	G	D		E	B	160	F	C		D	A	E		F#		G#		

			A♭		B♭		C	G	D	A		B	161	F		G	D	A	E		F#		G#			
			A♭		B♭		C	G	D	A	E	B	162	F	C	G	D	A	E		F#		G#			
			A♭		B♭	F			D		E	B	163	F	C		D			B	F#		G#			
			A♭		B♭	F			D	A		B	164	F		G	D			B	F#		G#			
			A♭		B♭	F			D	A	E	B	165	F	C	G	D			B	F#		G#			
			A♭		B♭	F		G			E	B	166	F	C			A		B	F#		G#			
			A♭		B♭	F		G		A	E	B	167	F	C	G		A		B	F#		G#			
			A♭		B♭	F		G	D		E	B	168	F	C		D	A		B	F#		G#			
			A♭		B♭	F		G	D	A		B	169	F		G	D	A		B	F#		G#			
			A♭		B♭	F		G	D	A	E	B	170	F	C	G	D	A		B	F#		G#			
			A♭		B♭	F	C			A	E	B	171	F	C	G			E	B	F#		G#			
			A♭		B♭	F	C		D		E	B	172	F	C		D		E	B	F#		G#			
			A♭		B♭	F	C		D	A	E	B	173	F	C	G	D		E	B	F#		G#			
			A♭		B♭	F	C	G			E	B	174	F	C			A	E	B	F#		G#			
			A♭		B♭	F	C	G		A	E	B	175	F	C	G		A	E	B	F#		G#			
			A♭		B♭	F	C	G	D		E	B	176	F	C		D	A	E	B	F#		G#			
			A♭		B♭	F	C	G	D	A		B	177	F		G	D	A	E	B	F#		G#			
			A♭		B♭	F	C	G	D	A	E	B	178	F	C	G	D	A	E	B	F#		G#			
22	Augmented ♭		A♭	E♭			C	G			E	B	179	F	C			A	E			C#	G#		Augmented #	21
			A♭	E♭			C	G		A	E	B	180	F	C	G		A	E			C#	G#			
			A♭	E♭			C	G	D	A	E	B	181	F	C	G	D	A	E			C#	G#			
			A♭	E♭		F		G		A	E	B	182	F	C	G		A		B		C#	G#			
			A♭	E♭		F		G	D		E	B	183	F	C		D	A		B		C#	G#			
			A♭	E♭		F		G	D	A	E	B	184	F	C	G	D	A		B		C#	G#			
			A♭	E♭		F	C			A	E	B	185	F	C	G			E	B		C#	G#			
			A♭	E♭		F	C		D	A	E	B	186	F	C	G	D		E	B		C#	G#			
			A♭	E♭		F	C	G		A	E	B	187	F	C	G		A	E	B		C#	G#			
			A♭	E♭		F	C	G	D		E	B	188	F	C		D	A	E	B		C#	G#			
			A♭	E♭		F	C	G	D	A	E	B	189	F	C	G	D	A	E	B		C#	G#			
24	Chromatic B♭ E♭ A♭		A♭	E♭	B♭			G	D	A	E	B	190	F	C	G	D	A			F#	C#	G#		Chromatic F# C# G#	23
			A♭	E♭	B♭		C		D	A	E	B	191	F	C	G	D		E		F#	C#	G#			
			A♭	E♭	B♭		C	G		A	E	B	192	F	C	G		A	E		F#	C#	G#			
			A♭	E♭	B♭		C	G	D	A	E	B	193	F	C	G	D	A	E		F#	C#	G#			
			A♭	E♭	B♭	F			D	A	E	B	194	F	C	G	D			B	F#	C#	G#			
			A♭	E♭	B♭	F		G	D	A	E	B	195	F	C	G	D	A		B	F#	C#	G#			
			A♭	E♭	B♭	F	C	G	D	A	E	B	196	F	C	G	D	A	E	B	F#	C#	G#			
26	Whole-Tone ♭	D♭		E♭		F		G		A		B	197	F		G		A		B		C#		D#	Whole-Tone #	25
		D♭		E♭		F		G		A	E	B	198	F	C	G		A		B		C#		D#		
		D♭		E♭		F		G	D		E	B	199	F	C		D	A		B		C#		D#		
		D♭		E♭		F		G	D	A	E	B	200	F	C	G	D	A		B		C#		D#		
		D♭		E♭		F	C		D		E	B	201	F	C		D		E	B		C#		D#		
		D♭		E♭		F	C		D	A	E	B	202	F	C	G	D		E	B		C#		D#		
		D♭		E♭		F	C	G		A	E	B	203	F	C	G		A	E	B		C#		D#		
		D♭		E♭		F	C	G	D	A	E	B	204	F	C	G	D	A	E	B		C#		D#		
28	Chromatic B♭ E♭ D♭	D♭		E♭	B♭		C		D	A	E	B	205	F	C	G	D		E		F#	C#		D#	Chromatic F# C# D#	27
		D♭		E♭	B♭		C	G		A	E	B	206	F	C	G		A	E		F#	C#		D#		
		D♭		E♭	B♭		C	G	D		E	B	207	F	C		D	A	E		F#	C#		D#		
		D♭		E♭	B♭		C	G	D	A	E	B	208	F	C	G	D	A	E		F#	C#		D#		
		D♭		E♭	B♭	F		G		A	E	B	209	F	C	G		A		B	F#	C#		D#		
		D♭		E♭	B♭	F		G	D	A	E	B	210	F	C	G	D	A		B	F#	C#		D#		
		D♭		E♭	B♭	F	C		D	A	E	B	211	F	C	G	D		E	B	F#	C#		D#		
		D♭		E♭	B♭	F	C	G	D	A	E	B	212	F	C	G	D	A	E	B	F#	C#		D#		
30	Octatonic ♭	D♭	A♭		B♭	F		G	D		E	B	213	F	C		D	A		B	F#		G#	D#	Octatonic #	29
		D♭	A♭		B♭	F		G	D	A	E	B	214	F	C	G	D	A		B	F#		G#	D#		
		D♭	A♭		B♭	F	C		D	A	E	B	215	F	C	G	D		E	B	F#		G#	D#		
		D♭	A♭		B♭	F	C	G	D	A	E	B	216	F	C	G	D	A	E	B	F#		G#	D#		

32	Chromatic E♭ A♭ D♭		D♭	A♭	E♭		F	C	G		A	E	B	217	F	C	G		A	E	B		C#	G#	D#		Chromatic C# G# D#	31
			D♭	A♭	E♭		F	C	G	D	A	E	B	218	F	C	G	D	A	E	B		C#	G#	D#			
34	Chromatic B♭ E♭ A♭ D♭		D♭	A♭	E♭	B♭		C	G	D	A	E	B	219	F	C	G	D	A	E		F#	C#	G#	D#		Chromatic F# C# G# D#	33
			D♭	A♭	E♭	B♭	F		G	D	A	E	B	220	F	C	G	D	A		B	F#	C#	G#	D#			
			D♭	A♭	E♭	B♭	F	C	G	D	A	E	B	221	F	C	G	D	A	E	B	F#	C#	G#	D#			
36	12-Tone ♭	G♭	D♭	A♭	E♭	B♭	F	C	G	D	A	E	B	222	F	C	G	D	A	E	B	F#	C#	G#	D#	A#	12-tone #	35

Figure 26-3. The duality of the Natural Harmonic Procession table with modalities

Character of Modalities

Some modalities are more readily perceived than others. For example, while the diatonic or Lydian character is easily recognized, the qualities associated with advanced dissonance or heightened chromaticism are more difficult to describe; in such cases, the generic term *Chromatic* is used (as in Chromatic F# C# G# D#).

Figure 26-4 details the chromatic content of the quintal prime forms and the numeric quintal prime forms of all modalities.

No.	Modality ♭	Span	Chromatic Content of the Quintal Prime Form	Chromatic Content of the Numeric Quintal Prime Form	Chromatic Content of the Quintal Prime Form	Modality #	No.
2	Suspended Triad ♭	3	–	–	–	Suspended Triad #	1
4	Quartal ♭	4	–	–	–	Quartal #	3
6	Pentatonic ♭	5	–	–	–	Pentatonic #	5
8	Ionian Hexachord ♭	6	–	–	–	Ionian Hexachord #	7
10	Diatonic ♭	7	–	–	–	Diatonic #	9
12	Mixolydian	8	B♭	8	F#	Lydian	11
14	Mystic	9	E♭	9	C#	Enigmatic	13
16	Blues ♭	9	B♭ E♭	8, 9	F# C#	Blues #	15
18	Diminished ♭	10	A♭	10	G#	Diminished #	17
20	Romanian	10	B♭ A♭	8, 10	F# G#	Hungarian	19
22	Augmented ♭	10	E♭ A♭	9, 10	C# G#	Augmented #	21
24	Chromatic B♭ E♭ A♭	10	B♭ E♭ A♭	8, 9, 10	F# C# G#	Chromatic F# C# G#	23
26	Whole-Tone ♭	11	E♭ D♭	9, 11	C# D#	Whole-Tone #	25
28	Chromatic B♭ E♭ D♭	11	B♭ E♭ D♭	8, 9, 11	F# C# D#	Chromatic F# C# D#	27
30	Octatonic ♭	11	B♭ A♭ D♭	8, 10, 11	F# G# D#	Octatonic #	29
32	Chromatic E♭ A♭ D♭	11	E♭ A♭ D♭	9, 10, 11	C# G# D#	Chromatic C#G# D#	31
34	Chromatic B♭ E♭ A♭ D♭	11	B♭ E♭ A♭ D♭	8, 9, 10, 11	F# C# G# D#	Chromatic F# C# G# D#	33
36	12-Tone ♭	12	12-Tone ♭	8,9,10,11,12	12-Tone #	12-Tone #	35

Figure 26-4. The duality of modalities in the Natural Harmonic Procession with the chromatic content of the quintal prime forms and the numeric quintal prime forms

In the final table (Figure 26-5), we offer a set of provisional descriptions for various modalities in terms of their character or emotional impact on the listener. This classification is not scientific—it is subjective and culturally conditioned—but it provides a useful starting point for exploring the expressive potential of each modality in programmatic narratives.

The Modal Harmonic Procession

The Modal Harmonic Procession is a variation of the Natural Harmonic Procession in which the sets advance by modalities rather than by individual spans. This modal version of the Procession is presented in **Tables 20–32** in the second half of the book.

Practical Applications of Modality Classification

The classification of modalities can assist the composer or theorist in the process of:

- Identifying a particular modality and locating all sets that belong to it
- Cross-referencing modalities in search of similarities (internal fractal recursions) among subsets and supersets
- Identifying sonorities that are unique to a given modality
- Isolating sonorities with a predetermined number of notes (for example, pentachords or hexachords) in order to identify the modalities that can be expressed through five- or six-voice harmony
- Identifying modalities suited to particular programmatic narratives
- Identifying modalities whose level of complexity is appropriate for a given context or ensemble

When harmonic complexity must be carefully managed—as in writing for chorus—the Diatonic modality and its closest relatives are often the safest choices. In such cases, the Harmonic Processions tables help define a limited and practical harmonic palette.

Conversely, when composers wish to expand their harmonic language and move beyond diatonicism, the Harmonic Processions tables offer a wealth of sonorities to explore.

No.	Modality	Projection	Span	Character
1	Suspended Triad #	#	3	Neutrality
2	Suspended Triad ♭	♭	3	Neutrality
3	Quartal #	#	4	Peace, rest, stasis, neutrality
4	Quartal ♭	♭	4	Peace, rest, stasis, neutrality
5	Pentatonic #	#	5	Bliss, peace, rest, beauty, stasis, neutrality, contentment
6	Pentatonic ♭	♭	5	Bliss, peace, rest, beauty, stasis, neutrality, contentment
7	Ionian Hexachord #	#	6	Bliss, peace, rest, beauty, stasis, neutrality, contentment
8	Ionian Hexachord ♭	♭	6	Bliss, peace, rest, beauty, stasis, neutrality, contentment
9	Diatonic #	#	7	Traditionalism, joy, sorrow, introspection
10	Diatonic ♭	♭	7	Traditionalism, joy, sorrow, introspection
11	Lydian	#	8	Curiosity, wonderment, amazement
12	Mixolydian	♭	8	Reserved curiosity, contentment, grounding
13	Enigmatic	#	9	Mystery, enchantment, curiosity,
14	Mystic	♭	9	Introspection, rumination, reset
15	Blues #	#	9	Edginess, brightness, boldness, confidence
16	Blues ♭	♭	9	Deep sorrow, grudge, complaint
17	Diminished #	#	10	Pain, sorrow, regret, disappointment, hurt, loneliness
18	Diminished ♭	♭	10	Pain, sorrow, regret, disappointment, hurt, loneliness
19	Hungarian	#	10	Expectation, tartness, brightness, edginess
20	Romanian	♭	10	Inevitability, tartness, darkness, edginess
21	Augmented #	#	10	Alarm, dread, danger, tragedy, horror
22	Augmented ♭	♭	10	Deep sorrow, mourning
23	Chromatic F# C# G#	#	10	Pain, instability
24	Chromatic B♭ E♭ A♭	♭	10	Pain, instability
25	Whole-Tone #	#	11	Transcendence, enchantment
26	Whole-Tone ♭	♭	11	Transcendence, enchantment
27	Chromatic F# C# D#	#	11	Tension, ambiguity, instability
28	Chromatic B♭ E♭ D♭	♭	11	Tension, ambiguity, instability
29	Octatonic #	#	11	Tension, ambiguity, instability
30	Octatonic ♭	♭	11	Tension, ambiguity, instability
31	Chromatic C#G# D#	#	11	Tension, ambiguity, instability
32	Chromatic E♭ A♭ D♭	♭	11	Tension, ambiguity, instability
33	Chromatic F# C# G# D#	#	11	Tension, ambiguity, instability
34	Chromatic B♭ E♭ A♭ D♭	♭	11	Tension, ambiguity, instability
35	12-Tone #	#	12	Tension, ambiguity, instability
36	12-Tone ♭	♭	12	Tension, ambiguity, instability

Figure 26-5. Programmatic classification of modalities

In Practice

- Study the following tables:
  - **Table 20** - The Modal Harmonic Procession of All Sets
  - **Table 21** - The Modal Harmonic Procession of All Sets, Including the Numeric Quintal and the Chromatic Prime Form
  - **Tables 22 - 32** - the Modal Harmonic Processions sorted by the number of notes

- In **Table 5** - The Natural Harmonic Procession of Sharp-Projecting Sets, locate all sets that contain the pentachord F–C–G–D–A. Then identify all of its other transpositions.

- If you were to compose a four-voice work for a college-level chorus, how far down the Natural Harmonic Procession would you extend your harmonic palette? Explain your reasoning.

- If you were to compose a six-voice work for a professional chorus that specializes in modern music, how far down the Natural Harmonic Procession would you extend your harmonic palette? Explain your reasoning.

- Write an 8-chord, 4-voice progression in the modality of your choice.

27. Modulation

Modulation is a harmonic motion from one area to another. This motion may proceed from one tonality to another through transposition while remaining within the same modality (for example, from Lydian to Lydian), or it may move through different modalities altogether.

The following example illustrates a modulation from the A♭-major triad, 8# (F 3-11B) to the G-major triad, 8# (F 3-11B). It employs transposition and remains within the same modality (Pentatonic #). Although the process takes place on the circle of fifths as a clockwise rotation, it is more straightforward to illustrate it on the matrix of fifths, moving from left to right (Figure 27-1).

HP	Forte	Modality	Matrix of Fifths											
8#	3-11B	Pentatonic #	D♭	**A♭**	**E♭**	B♭	F	**C**	G	D	A	E	B	F#
8#	3-11B	Pentatonic #	D♭	A♭	**E♭**	**B♭**	F	C	**G**	D	A	E	B	F#
8#	3-11B	Pentatonic #	D♭	A♭	E♭	**B♭**	**F**	C	G	**D**	A	E	B	F#
8#	3-11B	Pentatonic #	D♭	A♭	E♭	B♭	**F**	**C**	G	D	**A**	E	B	F#
8#	3-11B	Pentatonic #	D♭	A♭	E♭	B♭	F	**C**	**G**	D	A	**E**	B	F#
8#	3-11B	Pentatonic #	D♭	A♭	E♭	B♭	F	C	**G**	**D**	A	E	**B**	F#

Figure 27-1. Modulation from the A♭M triad to the GM triad, via transposition

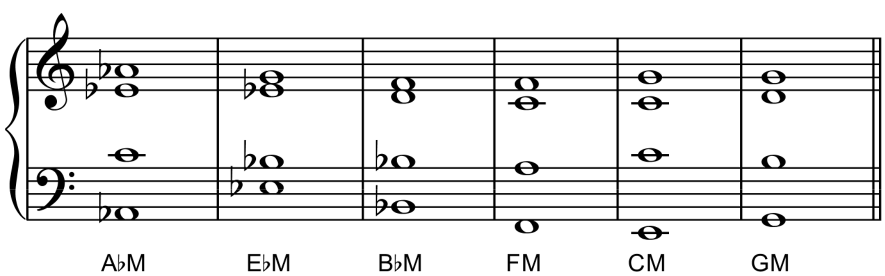


Figure 27-2. Modulation from the A♭M triad to the GM triad, via transposition, in music notation

The following example illustrates a modulation from E♭M7, 20S (F 4-20), to DM7, 20S (F 4-20), (Figure 27-3 and Figure 27-4). This modulation traverses multiple modalities and moves from one tonal area to another. Once again, notice the clockwise rotation—represented on the matrix of fifths as a shift from left to right. Although this modulation is less constrained than the previous one, the notes still migrate gradually toward the right side of the matrix in an organic but a directed manner.

To prepare this modulation, we begin by mapping the first and last chords. From there, we chart the gradual migration of the notes from the left side of the matrix to the right, planning for a variety of modalities if that is our objective. Throughout the process, we consult the Harmonic Processions tables to identify the quintal prime forms of the sets and their transpositions, ensuring that each step fits coherently within the matrix.

HP	Forte	Modality	Matrix of Fifths												
20S	4-20	Ionian Hexachord #/♭	A♭	**E♭**	**B♭**	F	C	**G**	**D**	A	E	B	F#	C#	G#
32S	4-21	Diatonic #/♭	A♭	**E♭**	B♭	**F**	C	**G**	D	**A**	E	B	F#	C#	G#
29♭	4-27A	Diatonic ♭	A♭	E♭	B♭	**F**	C	G	**D**	**A**	E	**B**	F#	C#	G#
78♭	4-19A	Mystic	A♭	E♭	B♭	**F**	C	G	**D**	**A**	E	B	F#	**C#**	G#
20S	4-20	Ionian Hexachord #/♭	A♭	E♭	B♭	F	C	G	**D**	**A**	E	B	**F#**	**C#**	G#

Figure 27-3. Modulation from E♭M7 to DM7, employing various modalities

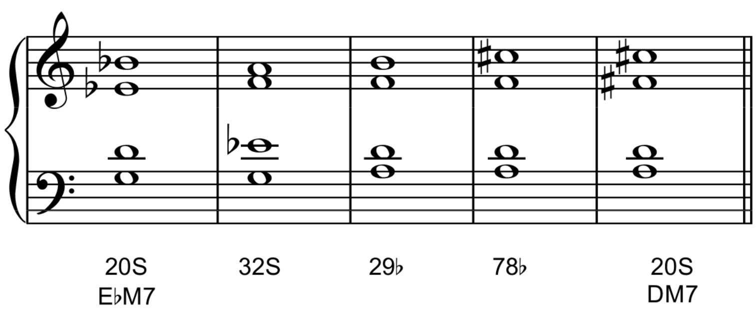


Figure 27-4. Modulation from E♭M7 to DM7, employing various modalities

The following modulation illustrates movement from one modality to another—from Lydian/Mixolydian to Quartal. In this example, the modulation proceeds from a wide span of 8 and contracts to a span of 4 (Figure 27-5 and Figure 27-6).

HP	Forte	Modality	Matrix of Fifths									
67S	4-9	Lydian/Mixolydian	E♭	**B♭**	**F**	C	G	D	A	**E**	**B**	F#
56#	4-5B	Lydian	E♭	**B♭**	**F**	C	G	D	**A**	E	**B**	F#
32S	4-21	Diatonic #/♭	E♭	**B♭**	F	**C**	G	**D**	A	**E**	B	F#
11S	4-26	Pentatonic #/♭	E♭	B♭	**F**	**C**	G	**D**	**A**	E	B	F#
6S	4-23	Quartal #/♭	E♭	B♭	**F**	**C**	**G**	**D**	A	E	B	F#

Figure 27-5. Modulation from the Lydian/Mixolydian to the Quartal modality, with span contraction

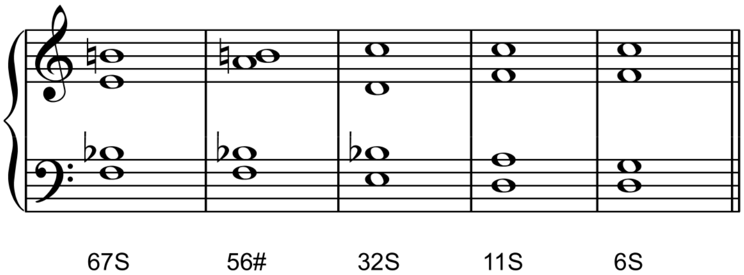


Figure 27-6. Modulation from to the Lydian/Mixolydian to the Quartal modality, with span contraction

The following modulation proceeds from a sharp-projecting tetrachord 16# (F 4-14B) to its flat-projecting mirror, 16♭ (F 4-14A), (Figure 27-7 and Figure 27-8).

HP	Forte	Modality	Matrix of Fifths
16#	4-14B	Ionian Hexachord #	B♭ **F** **C** **G** D A **E** B
17#	4-11B	Ionian Hexachord #	B♭ **F** **C** G **D** A **E** B
18S	4-10	Ionian Hexachord #/♭	B♭ **F** C **G** **D** A **E** B
17♭	4-11A	Ionian Hexachord ♭	B♭ **F** C **G** D **A** **E** B
16♭	4-14A	Ionian Hexachord ♭	B♭ **F** C G **D** **A** **E** B

Figure 27-7. Modulation from one modality to its mirror modality, within the same tonal area

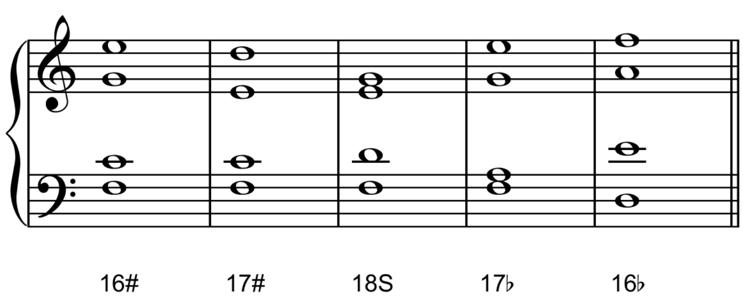


Figure 27-8. Modulation from one modality to its mirror modality, within the same tonal area

From the preceding examples we learn that modulation is an organic process—a movement from one tonality to another, or from one modality to another, or both—unfolding along the circle of fifths (illustrated here on the matrix of fifths). The approach does not fundamentally change whether the tonalities are closely or distantly related. Progressions between more distant tonalities may require a few additional steps, but the underlying procedure remains the same.

While tonic–dominant relationships, pivot chords, and leading tones do not need to be explicitly calculated, they nevertheless arise naturally within the process. And, of course, traditional voice-leading practices may be applied in music notation if the composer prefers to work within that framework.

In Practice

- On the matrix of fifths map out the following modulations, then write them out in keyboard notation:
  - B♭–C–G–A to G–D–B
  - E♭–B♭–G to A–E–C#–G#
  - D–A–F#–G# to B♭–F–D

28. The Natural Harmonic Procession and Dissonance Levels

As we examine the complexity and variety of sets in the Natural Harmonic Procession—with its multiple nested modalities—we cannot help but marvel at the elegance of a system not unlike the color spectrum or the artist's color wheel. The Natural Harmonic Procession orders harmonies from simple to complex in an objective sequence, but is it a reliable tool for measuring the dissonance of individual sets? The answer is both yes and no. Statistically, the simpler sets tend to be more consonant than the more complex ones, yet within the Natural Harmonic Procession there is no true, continuous gradient from consonance to dissonance.

For example, consider the whole-tone scale with the Natural Harmonic-Processions number 197S (F 6-35) (Figure 28-1). This bright, open hexachord is not as dissonant as several of the sets that precede it in the Procession. Compare it with a decachord whose Harmonic-Processions number is 196S (F 10-5) (Figure 28-2). Although its span is smaller than that of the hexachord, it contains four additional notes, and its interval-class content is far more robust, including four tritones.

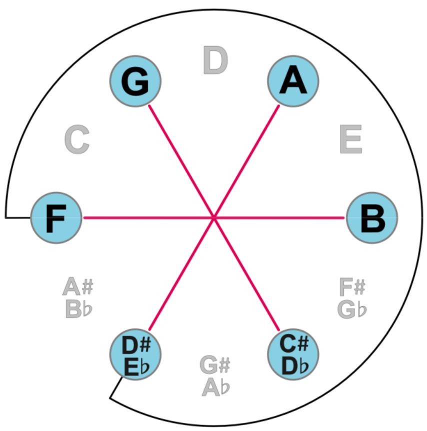


Figure 28-1. Whole-tone scale 197S (F 6-35), with the span of 11 and 3 tritones

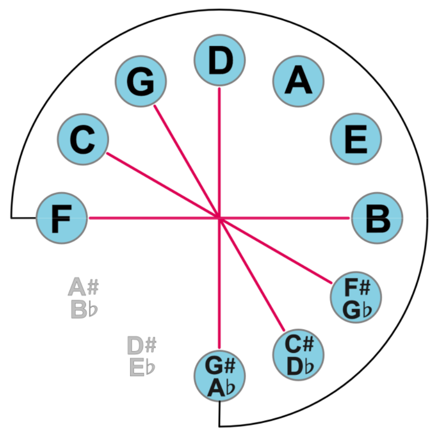


Figure 28-2. Decachord 196S (F 10-5) with the span of 10 and 4 tritones

We will discuss interval-class content, interval vectors, and precise dissonance levels in the chapters ahead. For now, it is important to re-emphasize that although the Natural Harmonic Procession approximates an ordering of sets according to their dissonance levels, its greatest strength lies in revealing and organizing the harmonic modalities.

In Practice

- Study the following tables and identify the points at which one modality ends and another begins. Observe the differences in complexity between neighboring sets.
  - **Table 2** - The Natural Harmonic Procession of All Sets
  - **Table 5** - The Natural Harmonic Procession of Sharp-Projecting Sets
  - **Table 6** - The Natural Harmonic Procession of Flat-Projecting Sets
  - **Table 20** - The Modal Harmonic Procession of All Sets

29. The Interval-Class Vector

The Limitations of the Natural Harmonic Procession

So far, in our discussion of the Natural Harmonic Procession, we have focused on the number of notes in a set, the span of the set, and its form. In essence, we have quantified each note in relation to the first note of the set—either F or B in the quintal prime form—as expressed through the Natural Harmonic Procession entry numbers (Chapter 22). Although this approach is sufficient for describing the unique shape and complexity of each set, there is yet a more thorough way to examine the harmonic relationships contained within.

We have also noted that while the Natural Harmonic Procession is best suited for showcasing and distinguishing among modalities, it is not designed to quantify levels of consonance or dissonance. For this reason, we will now shift our focus away from the Natural Harmonic Procession—and its derivative, the Modal Harmonic Procession—in order to examine the dissonance levels of all sets with greater precision.

The Interval-Class Vector

The concept of the interval-class vector was formalized by Donald Martino in 1961[25] and later popularized and systematized by Allen Forte in his 1973 book *The Structure of Atonal Music*[26]. Interval-class vectors for all sets have since been widely published across music-theory literature and remain an essential resource for both composers and theorists. Let us introduce this concept in a straightforward way.

In any given set, the notes form intervals not only with the first note of the set but with every other note as well, creating a unique harmonic footprint. This footprint can be inventoried and quantified by calculating the interval-class vector.

Let us calculate the interval-class vector for the F-major triad, 8# (F 3-11B). To do this, we must take an inventory of all intervals—and therefore all interval classes[27]—present in the set (Figure 29-1). We find the following:

1. M3 between F and A
2. m3 between A and C
3. P5 between F and C

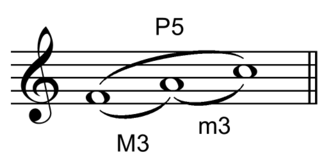


Figure 29-1. Inventory of intervals in the F-major triad

To inventory these intervals and interval classes, we first draw a table that accounts for all possible intervals and their inversions, from the smallest to the largest. Next, in each column, we record the number of intervals present using the binary system (Figure 29-2).

[25] Donald Martino, "The Source Set and Its Aggregate Formations," *Journal of Music Theory* 5, no. 2 (1961): 224–273.
[26] Allen Forte, *The Structure of Atonal Music* (New Haven: Yale University Press, 1973).
[27] Interval classes were discussed in Chapter 7.

m2 M7	M2 m7	m3 M6	M3 m6	P4 P5	TT
0	0	1	1	1	0

Figure 29-2. Inventory of interval classes in the F-major triad

Thus, the inventory of the interval classes present in the F-major triad can be expressed as a six-digit string: 0,0,1,1,1,0. In post-tonal music theory, this six-digit string is known as the interval-class vector.

Let us solidify this important concept by analyzing the interval-class vector of another set, 46# (F 4-Z15B), also known as the All-Interval tetrachord. This time, we will illustrate the interval classes on the circle of fifths using the interval-class signature[28] (Figure 29-3) and in a table (Figure 29-4).

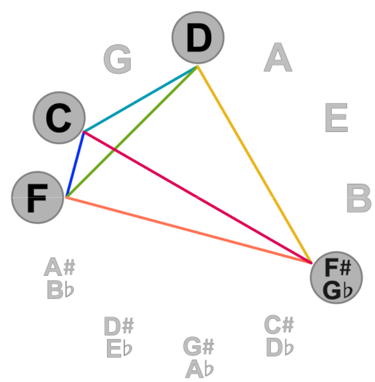


Figure 29-3. Inventory of interval classes of the All-Interval tetrachord, 46# (F 4-Z15B) using the interval-class signature

Here we find:

m2 M7	M2 m7	m3 M6	M3 m6	P4 P5	TT
1	1	1	1	1	1
F–F#	C–D	F–D	D–F#	F–C	C–F#

Figure 29-4. Inventory of interval classes of All-Interval tetrachord, 46# (F 4-Z15B)

When expressed as an interval-class vector, the set yields 1,1,1,1,1,1, hence the designation 'All-Interval tetrachord.' Containing exactly one instance of each interval class, it has proven remarkably versatile and has become a favorite among many composers.

[28] Interval-class signature is discussed in Chapter 33

In Practice

- Map the interval-class vector for the following sets. Begin with the circle of fifths and compile your results in the interval-class vector table:
  - Dominant 7th
  - Minor triad
  - Pentatonic scale

30. The Interval-Class Vector of the Diatonic Set

The Interval Class Inventory

Let us examine the interval-class vector of the diatonic set, 42S (F 7-35), using music notation. We take an inventory of the interval classes present, marking each with a slur (Figure 30-1).

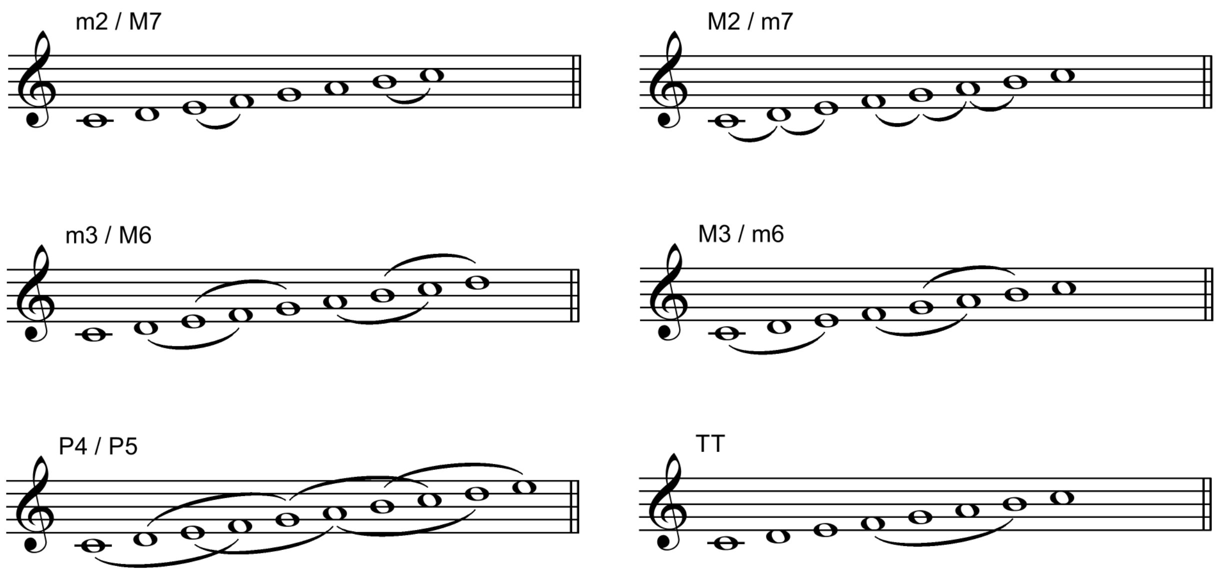
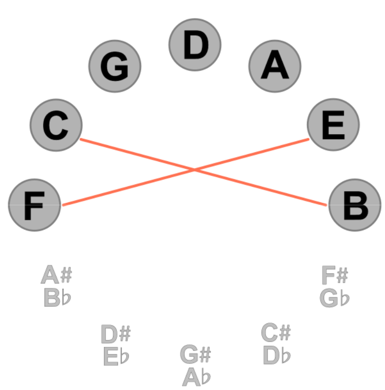
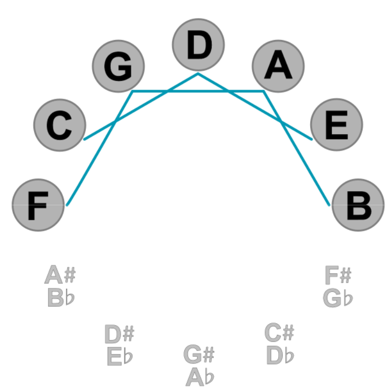


Figure 30-1. Inventory of interval classes in the diatonic set, in music notation

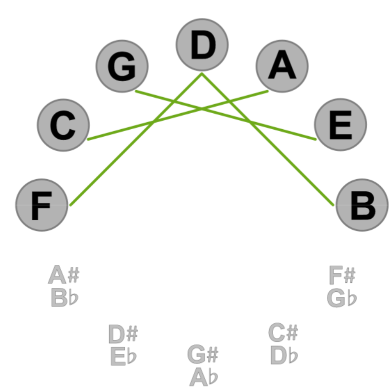
And here is the same inventory on the circle of fifths (Figure 30-2):



2 X m2



5 X M2



4 X m3

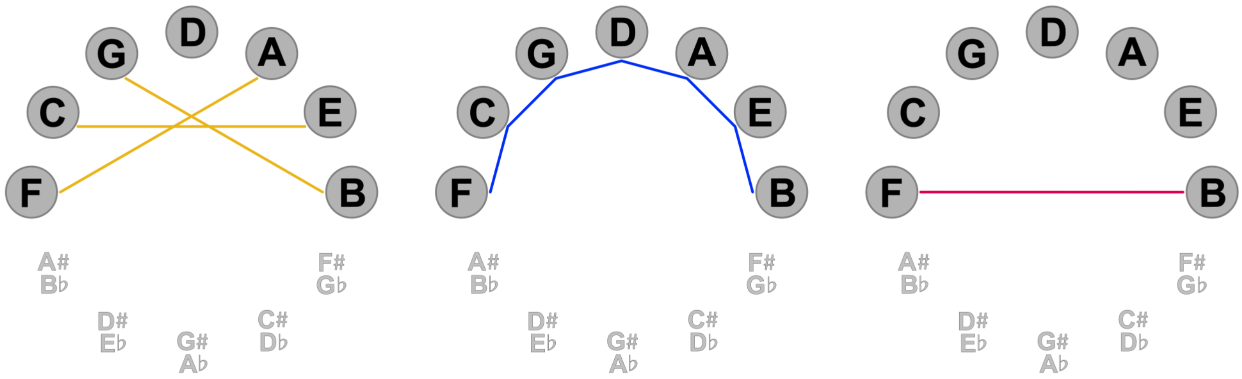


Figure 30-2. Inventory of interval classes in the diatonic set, on the circle of fifths

We find the following interval-class vector: 2,5,4,3,6,1 (Figure 30-3):

m2	M2	m3	M3	P4	TT
2	5	4	3	6	1

Figure 30-3. The interval-class vector of the diatonic set, in table format

The Ratios of the Interval Classes

As we inventory the interval classes of the diatonic set, we arrive at the interval-class vector 2,5,4,3,6,1. What is its significance?

The diatonic set is unique among all sets in that it presents all six interval classes—all possible distances on the circle of fifths—in the most compact configuration, that is, within the smallest possible span (Figure 30-4). This particular interval-class vector functions as a proportional model for harmonic organization, revealing the interval-class ratios that govern the entire circle of fifths.

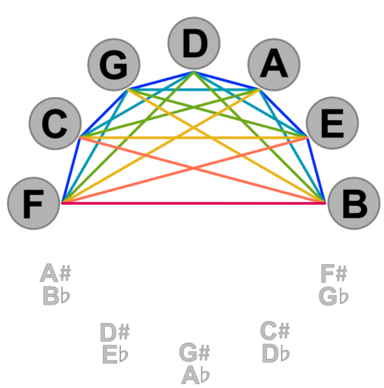


Figure 30-4. The interval classes of the diatonic set

The vector's interval classes occur in specific proportions, forming an equilibrium unique to the diatonic set. For the one tritone present, there are six perfect fourths, five major seconds, four minor thirds, three major thirds, and two minor seconds. These ratios constitute the consonance–dissonance gradient of the interval classes, based on their prevalence: the most consonant interval class is the most prevalent (6 × P4), and the most dissonant is the least prevalent (1 × TT) (Figure 30-5).

Inventory	Abbreviation	Interval Class	Consonance / Dissonance
6	P4	Perfect Fourth	Most consonant
5	M2	Major Second	↕
4	m3	Minor Third	
3	M3	Major Third	
2	m2	Minor Second	
1	TT	Tritone	Most dissonant

Figure 30-5. Interval classes in the diatonic set, ordered from most consonant to most dissonant

In Practice

- Take the inventory of all interval classes present in the Ionian hexachord F–C–G–D–A–E, 22#S (F 6-32), and compare it to the interval-class vector of the diatonic set.

31. The Interval-Class Dissonance Gradient and the Dissonance Curve

In Chapter 30, we took an inventory of all interval classes present in the diatonic set and arrived at the interval-class vector 2,5,4,3,6,1 (Figure 30-3). We also established that the diatonic set, in the proportions of its interval-class content, reveals an equilibrium of consonance and dissonance. The set contains, from the most consonant (and most prevalent) to the most dissonant (and least prevalent), the following interval classes: P4, M2, m3, M3, m2, TT.

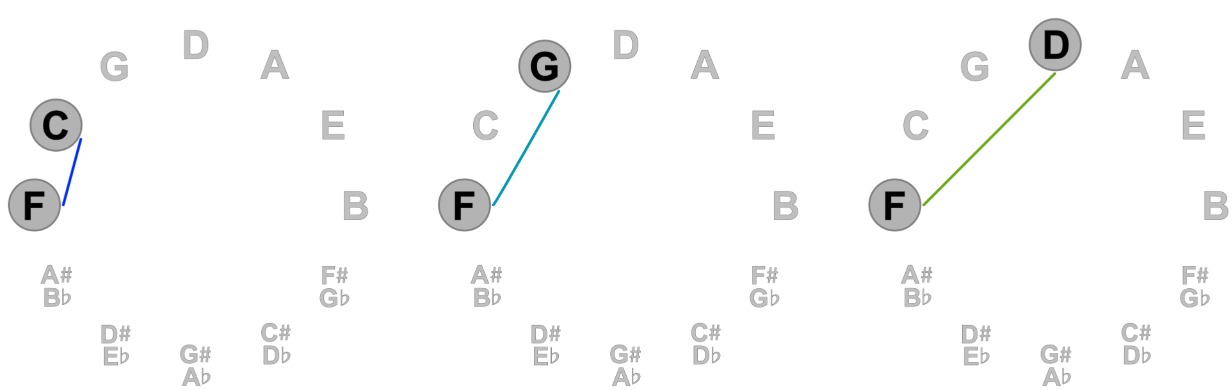
The Interval-Class Dissonance Gradient

Let us now present the same information in a table modeled on the traditional interval-class vector. While the interval-class vector is ordered chromatically—from the smallest interval class to the largest—our table is ordered from the most consonant and most prevalent interval class to the most dissonant and least prevalent. To distinguish this ordered string of values from the interval-class vector, we will refer to it as the interval-class dissonance gradient (Figure 31-1).

P4	M2	m3	M3	m2	TT
6	5	4	3	2	1

Figure 31-1. The interval-class dissonance gradient of the diatonic set

The interval-class dissonance gradient present in the diatonic set can also be observed in the geography of the circle of fifths. More consonant interval classes cluster tightly on the circle, while the more dissonant interval classes appear at progressively greater distances, following the same sequence as the interval-class gradient. Let us now illustrate the interval classes on the circle of fifths and observe how the levels of dissonance increase with each successive widening of the distance between the notes (Figure 31-2).



P4 #1,2 M2 #1,3 m3 #1,4

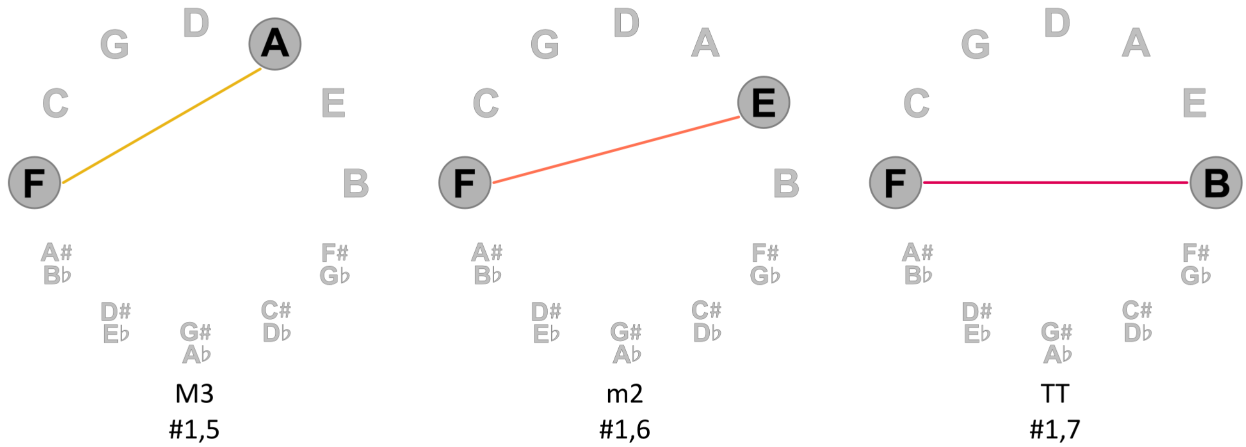


Figure 31-2. The interval-class dissonance gradient on the circle of fifths

Once we reach the diameter axis of the circle (F–B, the tritone), we have reached the furthest, most dissonant interval class. Continuing around the circle from that point would simply retrace the same interval classes in reverse order, moving through progressively diminishing levels of dissonance. The discussion of magnetism and repulsion in the circle of fifths in Chapter 14 has already introduced these principles.

The Dissonance Curve

Because consonance and dissonance in our model are not binary categories but rather values along a continuum, we can think of consonance as representing very low levels of dissonance, or of dissonance as very low levels of consonance. The journey from consonance to dissonance may be imagined as operating a dial whose setting increases or decreases the amount of dissonance present: turned down for consonance, turned up for dissonance.

This concept is analogous to measuring heat, where "cold" represents the absence or very low levels of heat, and the gradation from cold to hot reflects increasing levels of thermal energy (Figure 31-3). Similarly, on the interval-class dissonance gradient, the unison (or octave) contains no dissonance at all, the perfect fourth contains a small amount, the major third considerably more, and the tritone the most.

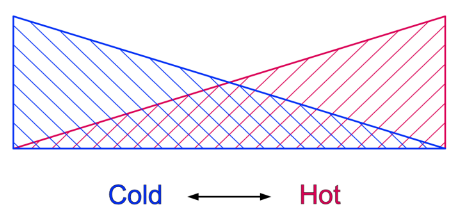


Figure 31-3. Gradation of temperature from cold to hot

However, the dissonance gradient of interval classes is not organized along a linear scale in the way temperature is. Instead, it follows a hyperbolic curve—a fragment of the reciprocal function **y = 1/x**, a special case of a rational function—in which **y** represents the dissonance value and **x** represents the prevalence of the six interval classes, from the least prevalent (the tritone) to the most prevalent (the perfect fourth) (Figure 31-4).

The values 1/6, 1/5, 1/4, 1/3, 1/2, and 1 on the dissonance scale (the y-axis) are derived directly from the interval-class vector of the diatonic scale, in which the proportions of the interval classes are expressed relative to the tritone. Thus, the ratio of perfect fourth to tritone is 6:1, of major second to tritone 5:1, of minor third to tritone 4:1, and so on. In other words, expressed informally, it takes six perfect fourths to balance the dissonance of a single tritone—one tritone is, proportionally speaking, six times more dissonant than one perfect fourth.

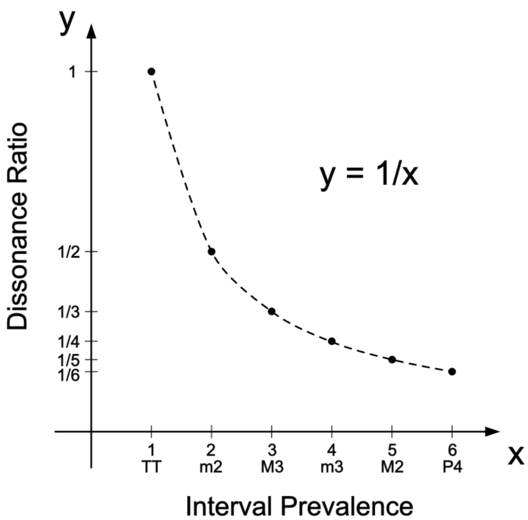


Figure 31-4. Dissonance gradient of interval classes quantified

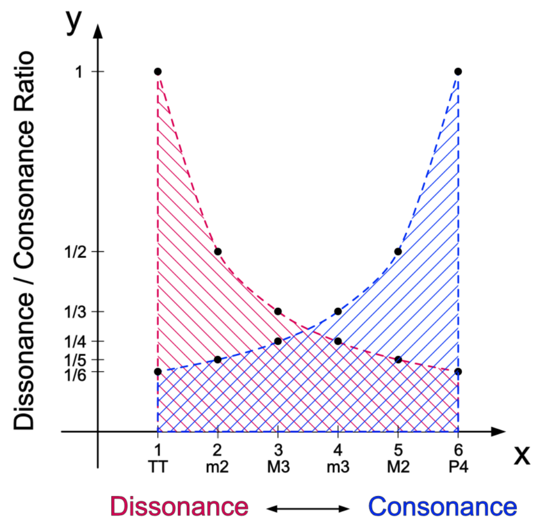


Figure 31-5. Dissonance gradient and consonance gradient juxtaposed

Conversely, if the dissonance gradient of the interval classes can be measured and expressed on the hyperbolic curve, so can the consonance gradient—although this one is inversely proportional to the dissonance gradient (Figure 31-5). However, for the ascending complexity of sets in the Harmonic Processions, it is preferable to adopt the hyperbolic scale of the dissonance curve and express complexity in terms of increasing levels of dissonance rather than decreasing levels of consonance.

Let us emphasize the significance of the dissonance curve and note that the difference in dissonance levels between the interval class of perfect fourth and the major second is minute. Likewise, the minor third and major third occupy essentially the same territory on the curve. By contrast, the minor second breaks sharply away from the preceding intervals, and the tritone leaves them all far behind.

Conversely, the same pattern appears in the consonance gradient: the difference in consonance levels between the tritone and the minor second is minimal, the minor and major thirds inhabit a closely related region, the major second exhibits strong consonance, and the perfect fourth surpasses all other interval classes.

In Practice

- Revisit the assignment from Chapter 13, in which you were asked to subjectively order the intervals from most consonant to most dissonant. Compare your original perceptions with the presented theory.

- Examine **Table 4** - The Natural Harmonic Procession of All Sets, Including Interval-Class Dissonance Gradients.

32. Quantifying Dissonance Levels of Sets and the Dissonance-Gradient Harmonic Procession

Quantifying Dissonance through the Interval-Class Dissonance Gradient

The dissonance level of a set can be quantified by examining its interval-class dissonance gradient and multiplying the number of each interval class present by its inherent ratio of dissonance. The following table lists these ratios and their decimal equivalents (Figure 32-1).

Interval Class	Dissonance Level as a Ratio	Dissonance Level as a Decimal
P4	1/6	0.166
M2	1/5	0.20
m3	1/4	0.25
M3	1/3	0.333
M2	1/2	0.50
TT	1	1.00

Figure 32-1. Dissonance level ratios of interval classes

Let us now calculate the dissonance level of a major triad, 8# (F 3-11B), whose interval-class gradient is 1,0,1,1,0,0 (1 P4, 1 m3, 1 M3). Using mathematical calculation, we get:

1/6 + 1/4 + 1/3 = 3/4 = **0.75**

Now let us do the same for the diatonic set, 42S (F 7-35), whose interval-class gradient is 6,5,4,3,2,1:

6 x 1/6 + 5 x 1/5 + 4 x 1/4 + 3 x 1/3 + 2 x 1/2 + 1 = **6**

And finally, for the twelve-tone set (the chromatic scale), 222S (F 12-1), whose interval-class gradient is 12,12,12,12,12,6:

12 x 1/6 + 12 x 1/5 + 12 x 1/4 + 12 x 1/3 + 12 x 1/2 + 6 x 1 = **23.40**

Dissonance Level as Percentage

Since the twelve-tone set (the chromatic scale) is the most complex and most dissonant set of all, its dissonance level of 23.40 represents the highest possible value and therefore serves as the standard of total dissonance at 100%. When we compare all other sets to this benchmark, their dissonance levels can be expressed as percentages of the chromatic value. Thus, the major triad has a dissonance percentage of **3.21%** (0.75 / 23.40), and the diatonic scale **25.64%** (6 / 23.40).

The Dissonance-Gradient Harmonic Procession

Dissonance levels, expressed as percentages of the twelve-tone scale's 100% dissonance value, are listed for all sets within the Harmonic Processions. A sequence of sets ordered according to ascending dissonance levels forms the Dissonance-Gradient Harmonic Procession (see **Tables 33–45** in the second half of the book). This Procession serves as a practical tool for monitoring dissonance levels when creating and resolving harmonic tension.

Interval-Class Dissonance Gradient Summary

To summarize the concept of the interval-class dissonance gradient, the theory of Harmonic Processions posits that:

1. Consonance and dissonance form a continuum, not a binary opposition, and operate on a hyperbolic gradient rather than a linear scale.

2. Interval classes follow a gradient from least dissonant to most dissonant: P4, M2, m3, M3, m2, TT.

3. Dissonance levels of interval classes can be quantified based on their proportions within the diatonic set and their positions on the circle of fifths, both of which reflect the same underlying gradient.

4. The only truly consonant interval class is the unison (interval of the octave); all other interval classes fall along the dissonance gradient. Interval classes traditionally perceived as "consonant" are understood here as those with lower levels of dissonance, not as categorically consonant.

5. Because dissonance can be quantified for interval classes, it can therefore be quantified for all sets, and these values—expressed as percentages relative to the twelve-tone set—form the Dissonance-Gradient Harmonic Procession, an ordering of sets by ascending dissonance.

In Practice

- Study **Table 33** – The Dissonance-Gradient Harmonic Procession of All Sets, and the subsequent tables, up to **Table 45**.

- Revisit the earlier tables and observe the dissonance levels listed in each. How do these levels relate to different spans and different modalities?

- Identify the level of momentary dissonance you find comfortable/tolerable—either in your own creative work or in the music of other composers. Specify the percentage.

- Identify the level of prolonged dissonance you find comfortable/tolerable—again, in your own output or in the work of others. Specify the percentage.

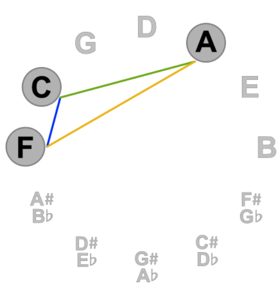
33. The Interval-Class Signature

While the interval-class dissonance gradient can be represented as a string of numbers or as a percentage, it becomes far more vivid when mapped onto the circle of fifths. We will call such a mapping the interval-class signature. The interval-class signature illustrates all intervallic relationships among the notes of a set and reflects both the number and the size of its interval classes. In this sense, the interval-class signature expresses the unique footprint—or DNA—of each set.

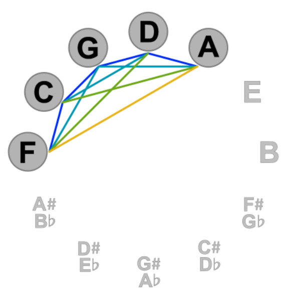
Below are interval-class signatures of selected sets. With the use of color, we can highlight the levels of dissonance present, as shown in the legend in Figure 33-1.

P4	M2	m3	M3	m2	TT

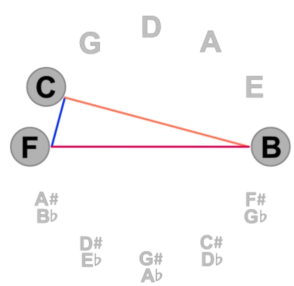
Figure 33-1. Colors assigned to interval classes



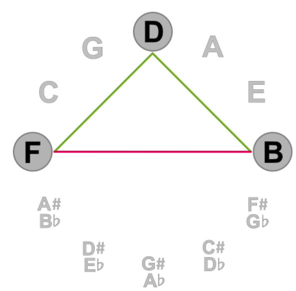
8♯ (F3-11B)
Major triad



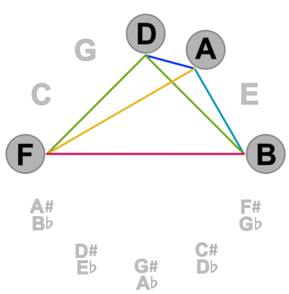
12♯S (F 5-35)
Pentatonic scale



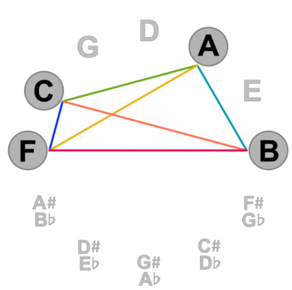
24♯ (3-5A)
Viennese trichord



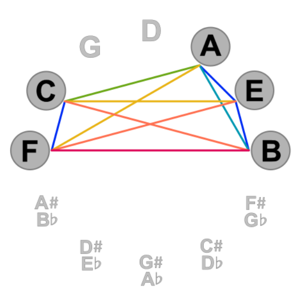
27S (F 3-10)
Diminished triad



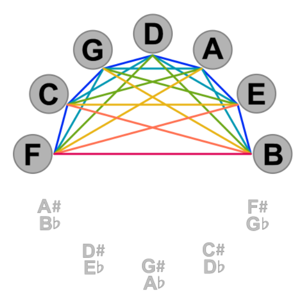
29♭ (F 4-27A)
Tristan chord



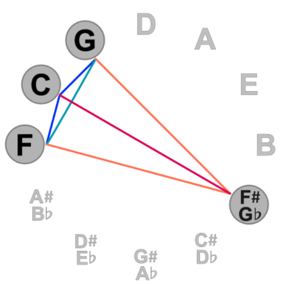
31♯ (F 4-Z29B)
All-Interval tetrachord



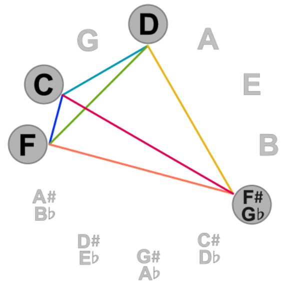
38♭ (F 5-20B)
Hirajōshi scale



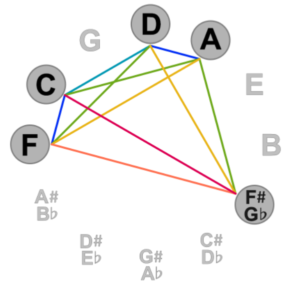
42S (F 7-35)
Diatonic scale



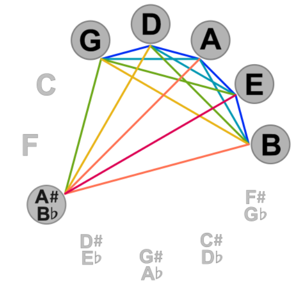
44#S (F 4-6)
Dream chord



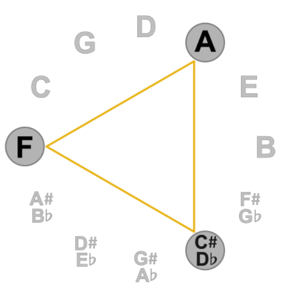
46# (F 4-Z15B)
All-Interval tetrachord



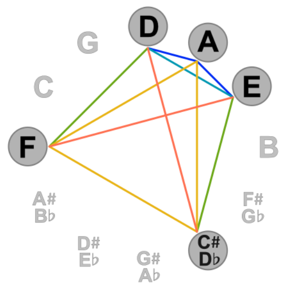
53# (F 5-32B)
Elektra chord



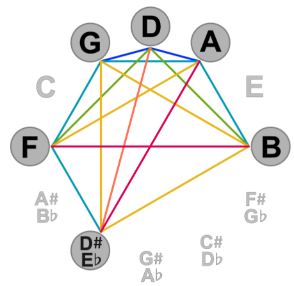
55♭ (F 6-Z47B)
6-Note Blues scale



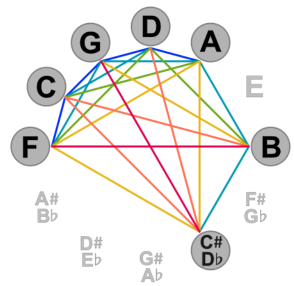
77#S (F 3-12)
Augmented triad



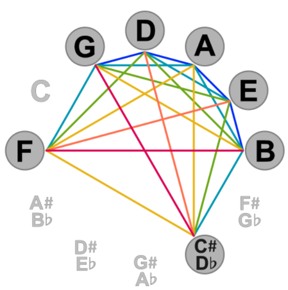
93#S (F 5-Z17)
Farben chord



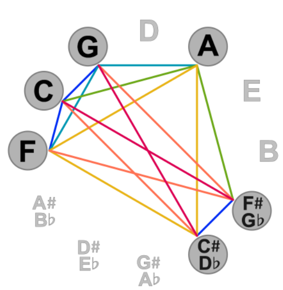
106♭ (F 6-34A)
Prometheus chord



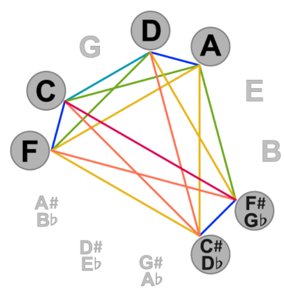
107# (F 7-24B)
Enigmatic



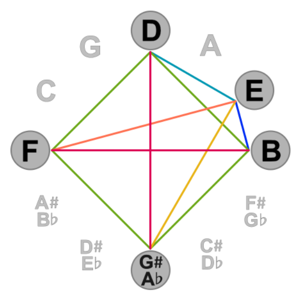
116#S (F 7-34)
Half-diminished scale



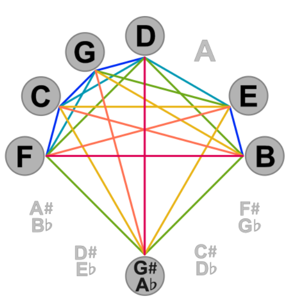
120# (F 6-Z17A)
All-trichord hexachord



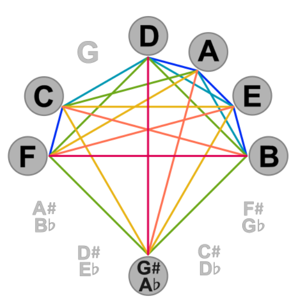
121# (F 6-Z44A)
Schoenberg hexachord



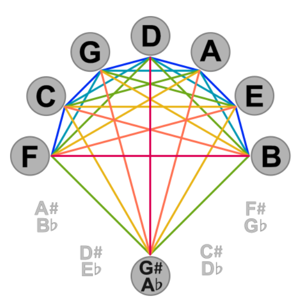
136♭ (F 5-31B)
Beta chord



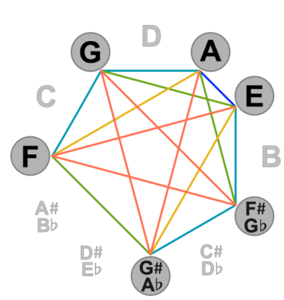
146# (F 7-32B)
Harmonic major scale



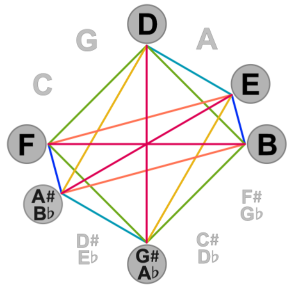
146♭ (F 7-32A)
Harmonic minor scale



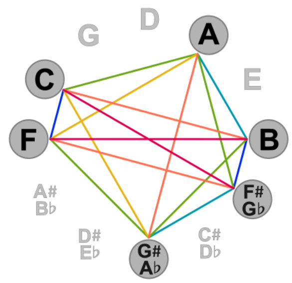
148S (F 8-26)
Bebop major scale



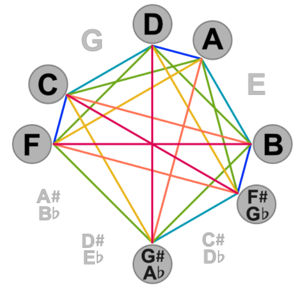
158#S (F 6-1)
Chromatic hexachord



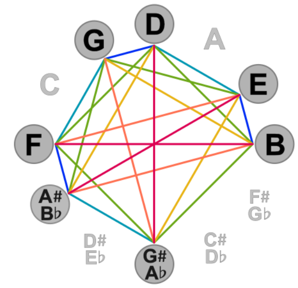
163♭ (F 6-30B)
Petrushka chord



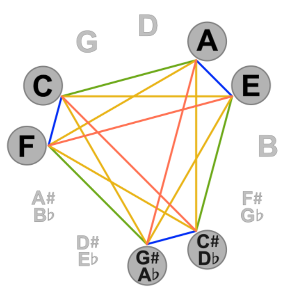
166#S (F 6-Z13)
Istrian scale



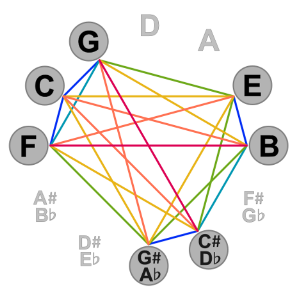
168# (F 7-31A)
Hungarian major scale



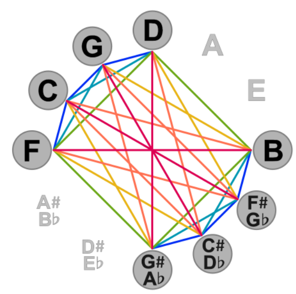
168♭ (F 7-31B)
Romanian major scale



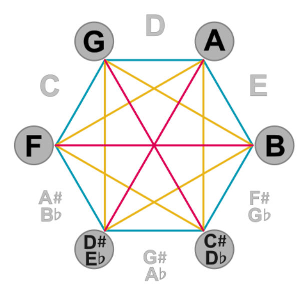
179#S (F 6-20)
“Ode-to-Napoleon” chord



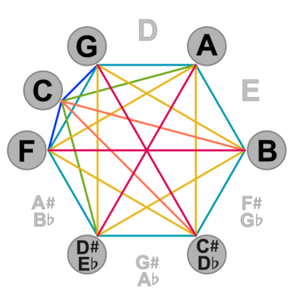
185#S (F 7-22)
Double-Harmonic scale



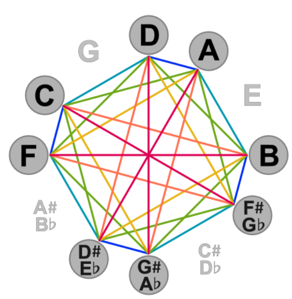
194#S (F 8-9)
Messiaen’s 4th mode



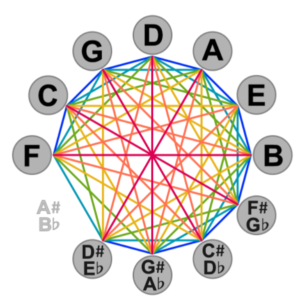
197S (F 6-35)
Whole-tone scale



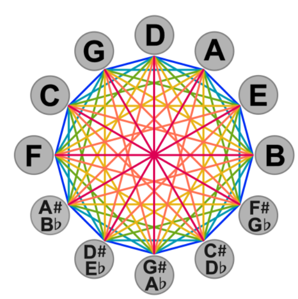
198#S (F 7-33)
Major Locrian scale



213#S (F 8-28)
Octatonic scale



221#S (F 11-1)
Northern Lights chord



222 (F 12-1)
Chromatic scale

In Practice

- From the Harmonic Processions tables, select three sets of varying complexities that have not been mapped in the examples above. Draw each set on the circle of fifths along with its interval-class signature.

34. Building and Resolving Harmonic Tension

Monitoring Dissonance Levels

Now that we have practiced modulations in Chapter 27 and quantified the dissonance levels of sets in Chapter 32, we can combine these ideas to create and resolve harmonic tension with greater precision. Once again, we will use the matrix of fifths, but this time we will choose sets for our chord progressions more purposefully, monitoring the dissonance-level percentages of each set. These percentages can be found in any of the tables in the second half of the book.

Let us begin by resolving the tension of the most dissonant hexachord, Messiaen's 5th mode, 129S (F 6-7), by progressing to a major triad, 8# (F 3-11B), ensuring that the dissonance percentages decrease with each successive chord. We will accomplish this in five steps (five sets), as shown in Figure 34-1 and Figure 34-2.

HP	Forte	Numeric Quintal Prime Form	Matrix of Fifths	Dissonance Level %
129S	6-7	1,2,3,7,8,9	A♭ **E♭ B♭ F** C G D **A E B** F#	28.77
74S	6-Z38	1,2,3,6,7,8	A♭ E♭ **B♭ F C** G D **A E B** F#	25.57
30#	5-29A	#1,2,3,4,7	A♭ E♭ B♭ **F C G D** A E **B** F#	13.82
11S	4-26	1,2,4,5	A♭ E♭ B♭ F **C G** D **A E** B F#	5.84
8#	3-11B	#1,2,5	A♭ E♭ B♭ F **C G** D A **E** B F#	3.21

Figure 34-1. Decreasing dissonance levels in a progression

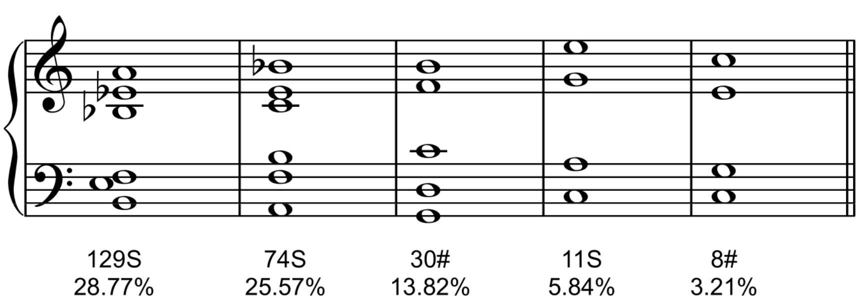


Figure 34-2. Decreasing dissonance levels in a progression

Now, beginning with a major triad on A♭, 8# (F 3-11B), we will build harmonic tension by increasing the dissonance level until we reach the hexachord 129S (F 6-7), and then resolve that tension toward the A-major triad, 8# (F 3-11B). The progression will consist of eight sets, with the most dissonant chord (129S) placed as the fourth set in the sequence (Figure 34-3 and Figure 34-4).

HP	Forte	Numeric Quintal Prime Form	Matrix of Fifths													Dissonance Level %
8#	3-11B	#1,2,5	D♭	**A♭**	**E♭**	B♭	F	**C**	G	D	A	E	B	F#	C#	3.21
21#	5-27A	#1,2,3,5,6	D♭	A♭	**E♭**	**B♭**	**F**	C	**G**	**D**	A	E	B	F#	C#	10.97
40♭	6-Z25B	♭1,2,3,4,6,7	D♭	A♭	**E♭**	**B♭**	F	**C**	**G**	**D**	**A**	E	B	F#	C#	20.01
129S	6-7	1,2,3,7,8,9	D♭	A♭	**E♭**	**B♭**	**F**	C	G	D	**A**	**E**	**B**	F#	C#	28.77
91#	5-26B	#1,3,5,6,9	D♭	A♭	E♭	B♭	**F**	C	**G**	D	**A**	**E**	B	F#	**C#**	15.24
34♭	5-25B	♭1,2,4,5,7	D♭	A♭	E♭	B♭	F	**C**	G	**D**	**A**	E	**B**	**F#**	C#	14.17
17#	4-11B	#1,2,4,6	D♭	A♭	E♭	B♭	F	C	G	**D**	**A**	E	**B**	F#	**C#**	7.05
8#	3-11B	#1,2,5	D♭	A♭	E♭	B♭	F	C	G	D	**A**	**E**	B	F#	**C#**	3.21

Figure 34-3. Increasing and decreasing dissonance levels in a progression

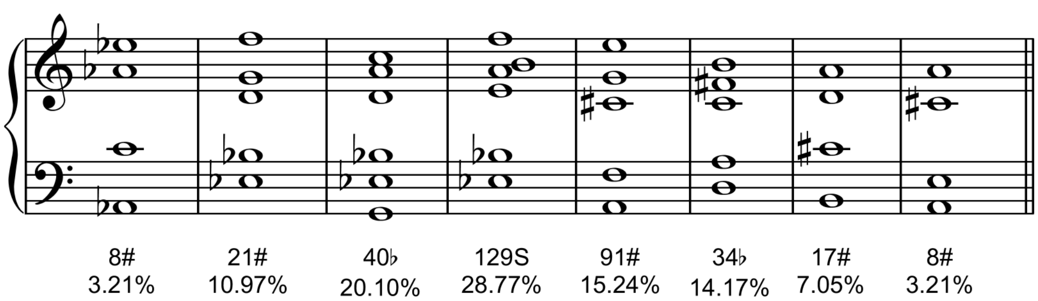


Figure 34-4. Increasing and decreasing dissonance levels in a progression

Increasing Harmonic Tension with Polarity Shift

While we can build harmonic tension by increasing the dissonance level, we can also heighten it by shifting polarities on the circle of fifths. Such a shift has a startling effect and produces a momentary sense of harmonic disorientation. Let us try this on a basic cadence (tonic–subdominant–dominant–tonic): C-major, F-major, G-major, and C-major. All three chords share the same dissonance level of 3.21%. The slight harmonic tension arises only from small shifts within the same hemisphere of the circle of fifths; in fact, the farthest any of these sets travel around the circle is two perfect fifths (F–C–G), as shown in Figure 34-5 and Figure 34-6.

HP	Forte	Matrix of Fifths									Dissonance Level %
8#	3-11B	B♭	F	**C**	**G**	D	A	**E**	B	F#	3.21
8#	3-11B	B♭	**F**	**C**	G	D	**A**	E	B	F#	3.21
8#	3-11B	B♭	F	C	**G**	**D**	A	E	**B**	F#	3.21
8#	3-11B	B♭	F	**C**	**G**	D	A	**E**	B	F#	3.21

Figure 34-5. Cadence within the hemisphere bounds

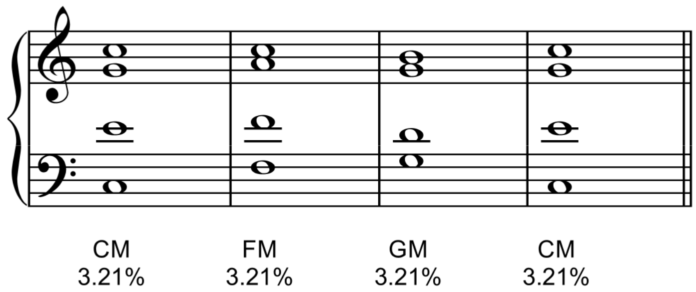


Figure 34-6. Cadence within the hemisphere bounds

Now let us use the same progression, but this time, after the F-major chord, we will introduce a D♭-major chord. Once again, all chords share the same dissonance level of 3.21%, yet the tension is heightened by the startling effect of polarity shifting. The D♭-major chord resides in the opposing hemisphere, five perfect fifths away from C (see Figure 34-7 and Figure 34-8). When voiced in first inversion, this striking sonority is interpreted as the Neapolitan chord.

HP	Forte	Matrix of Fifths												Dissonance Level %	
8#	3-11B	G♭	D♭	A♭	E♭	B♭	F	**C**	**G**	D	A	**E**	B	F#	3.21	
8#	3-11B	G♭	D♭	A♭	E♭	B♭	**F**	**C**	G	D	**A**	E	B	F#	3.21	
8#	3-11B	G♭	**D♭**	**A♭**	E♭	B♭	**F**	C	G	D	A	E	B	F#	3.21	**Polarity shift**
8#	3-11B	G♭	D♭	A♭	E♭	B♭	F	C	**G**	**D**	A	E	**B**	F#	3.21	
8#	3-11B	G♭	D♭	A♭	E♭	B♭	F	**C**	**G**	D	A	**E**	B	F#	3.21	

Figure 34-7. Cadence with a polarity shift

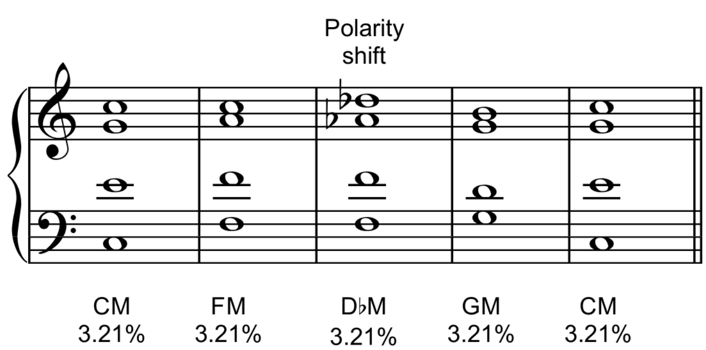


Figure 34-8. Cadence with a polarity shift

The polarity shift can function as a powerful tool on its own or in combination with increasing the dissonance levels. In the following example, the fourth chord in the progression carries the highest dissonance level and also ventures into the opposing hemisphere (Figure 34-9 and Figure 34-10).

HP	Forte	Matrix of Fifths	Dissonance Level %	
6S	4-23	G♭ D♭ A♭ E♭ B♭ **F** **C** **G** **D** A E B F# C#	4.91	
12S	5-35	G♭ D♭ A♭ E♭ B♭ F **C** **G** **D** **A** **E** B F# C#	8.97	
38♭	5-20B	G♭ D♭ A♭ E♭ B♭ **F** **C** G D **A** **E** **B** F# C#	15.46	
68#	5-7A	**G♭** **D♭** **A♭** E♭ B♭ F **C** **G** D A E B F# C#	19.37	**Polarity shift**
85#	5-Z18A	G♭ D♭ A♭ E♭ B♭ **F** **C** **G** D A **E** B F# **C#**	15.81	
35S	5-34	G♭ D♭ A♭ E♭ B♭ F C **G** D **A** **E** **B** F# **C#**	13.25	
18S	4-10	G♭ D♭ A♭ E♭ B♭ F C **G** D **A** **E** B **F#** C#	6.70	
8#	3-11B	G♭ D♭ A♭ E♭ B♭ F C G **D** **A** E B **F#** C#	3.21	

Figure 34-9. Increasing dissonance levels combined with a polarity shift

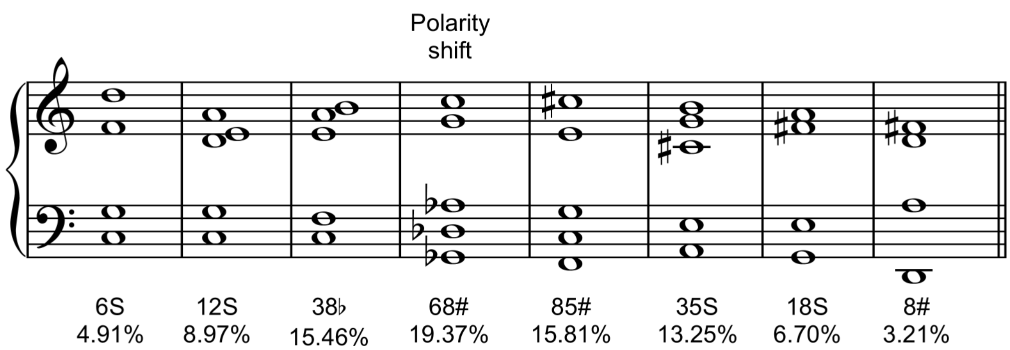


Figure 34-10. Increasing dissonance levels combined with a polarity shift

In Practice

Prepare the following progressions on the matrix of fifths and in music notation:

- Beginning with the A-minor triad, 8♭ (F 3-11A), write a progression that gradually increases in dissonance level until it reaches the pentachord 98S (F 5-15), and then gradually decreases in dissonance level to the F minor triad, 8♭ (F 3-11A). The progression should contain eight sets in total, with pentachord 98S placed as the fourth set. Use polarity shift to heighten its harmonic tension.

- Beginning with the whole-tone hexachord, 197S (F 6-35), build a progression of seven sets that gradually decrease in dissonance level, arriving at a major triad, 8# (F 3-11B). Apply polarity shift to the final set to create a startling resolution.

- Build a progression of eight sets beginning and ending with the pentachord 98S (F 5-15). The final pentachord should lie a tritone away from the first. The gradual decrease in dissonance level from the initial pentachord should lead to a major triad, 8# (F 3-11B), as the fifth set, from which a gradual increase in dissonance level should lead to the final pentachord.

35. Tonal Ambiguity

Avoiding Tonality

Although all the preceding tools point toward creating order and logic in harmonic progressions, the theory of Harmonic Processions does not imply that the tonal or consonant model is superior, nor that modulations must follow a particular pattern, nor that dissonance must be resolved in any prescribed way—or that it should be resolved at all. The Harmonic Processions theory can be equally useful in building and resolving harmonic tension by establishing tonal centers, as in maintaining harmonic ambiguity by avoiding any suggestion of tonality.

Let us build a progression that avoids diatonicism and consonance. Once again, we will use the matrix of fifths, but this time we will deliberately avoid emphasizing any tonal centers—that is, we will avoid clusters that promote consonance. If we do choose clusters, we will ensure that they are neutralized by notes in diametrical opposition (in the opposing hemisphere on the circle of fifths). To increase the harmonic complexity, we will work with hexachords (Figure 35-1 and Figure 35-2).

HP	Forte	Matrix of Fifths											
141♭	6-27B	A♭	**E♭**	B♭	F	**C**	G	D	**A**	**E**	B	**F#**	**C#**
151S	6-Z49	**A♭**	E♭	**B♭**	F	C	**G**	**D**	A	**E**	**B**	F#	C#
197S	6-35	A♭	**E♭**	B♭	**F**	C	**G**	D	**A**	E	**B**	F#	**C#**
197S	6-35	**A♭**	E♭	**B♭**	F	**C**	G	**D**	A	**E**	B	**F#**	C#
110#	6-Z3A	A♭	**E♭**	**B♭**	F	**C**	G	**D**	**A**	E	B	F#	**C#**
74S	6-Z38	**A♭**	**E♭**	**B♭**	F	C	G	D	**A**	**E**	**B**	F#	C#
158S	6-1	A♭	**E♭**	B♭	**F**	C	**G**	**D**	A	**E**	B	**F#**	C#
154♭	6-21A	**A♭**	E♭	**B♭**	F	**C**	G	**D**	A	E	**B**	**F#**	C#

Figure 35-1. A progression avoiding consonance or tonal centers, using hexachords

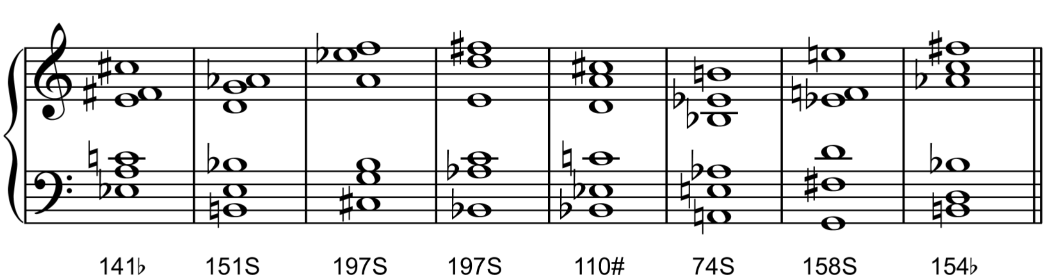


Figure 35-2. A progression avoiding consonance or tonal centers, using hexachords

And now, let us construct another matrix of harmonic ambiguity, but this time using only triads (Figure 35-3 and Figure 35-4).

HP	Forte	Matrix of Fifths											
45♭	3-3A	A♭	**E♭**	B♭	F	**C**	G	D	A	E	**B**	F#	C#
25♭	3-8B	**A♭**	E♭	B♭	F	C	G	**D**	A	E	B	**F#**	C#
14♭	3-4A	A♭	E♭	**B♭**	**F**	C	G	D	A	E	B	**F#**	C#
77S	3-12	A♭	E♭	B♭	**F**	C	G	D	**A**	E	B	F#	**C#**
8#	3-11B	A♭	E♭	B♭	F	**C**	**G**	D	A	**E**	B	F#	C#
45♭	3-3A	A♭	E♭	**B♭**	F	C	G	**D**	A	E	**B**	F#	C#
27S	3-10	A♭	**E♭**	B♭	F	**C**	G	D	**A**	E	B	F#	C#
43S	3-1	**A♭**	E♭	B♭	F	C	**G**	D	A	E	B	**F#**	C#

Figure 35-3. A progression avoiding consonance or tonal centers, using triads

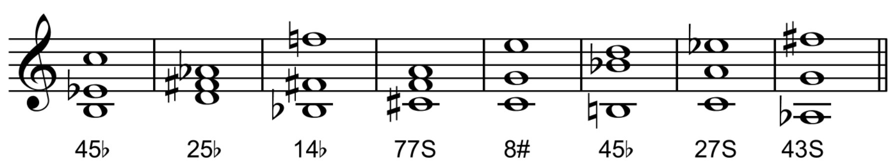


Figure 35-4. A progression avoiding consonance or tonal centers, using triads

Composing with Large Sets

In the previous chapters, we used various harmonic progressions to illustrate modulations and the shaping of harmonic tension. However, when harmonic ambiguity is desired, it is often advantageous not to modulate or to fluctuate the harmonic tension at all. Instead, one can avoid tonal or diatonic (consonant) allusions by composing or improvising within a large set. Nonachords, decachords, undechachords, and the dodecachord are particularly well suited for this purpose. Any of these sets provide ample melodic and harmonic material outside of diatonicism, offering endless variety without the need to modulate, yet they differ enough from one another that experienced composers can perceive their distinct harmonic inflections. The Harmonic Processions tables in the second half of the book include all of these sets in various groupings.

In Practice

- On the matrix of fifths and in keyboard notation, using only tetrachords or pentachords, prepare a progression that is tonally ambiguous.

36. Z-Relation and Dissonance Level

37. Modal Heptachords, Hexachords, and Pentachords

38. Tritone Substitution

This content is available exclusively
in the printed edition, which may be purchased
at www.DosiaMcKay.com

39. The Forte Numbers

In 1973, American music theorist and musicologist Allen Forte, in his book The Structure of Atonal Music[31], introduced a catalog of all possible pitch-class sets. He labeled each set with two numbers separated by a hyphen—for example, 4-1, 6-3, or 7-35—where the first number signifies the number of notes in the set, and the second number indicates its position within that group, ordered by the compactness of the set's chromatic prime form[32] which in his system always begins on C.

Let us examine the tetrachord F#–A–C–E#, which corresponds to Forte number 4-18A[33]. Because the sonority contains four pitch classes, it is labeled with the initial '4,' and the suffix '18A' identifies its position within the catalog of tetrachords based on the compactness of its chromatic prime form [0, 1, 4, 7]. This chromatic prime form is equivalent to the set class represented by C–C#–E–G derived in the process illustrated in Figure 39-1 through Figure 39-3.

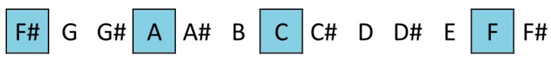


Figure 39-1. Tetrachord F#–A–C–E# on the chromatic scale

Figure 39-2. Tetrachord F#–A–C–E# on the chromatic scale, in the most compact voicing possible

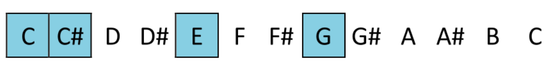


Figure 39-3. Tetrachord F#–A–C–E# in its chromatic prime form (C–C#–E–G), transposed to begin on C

Since Forte numbers are based on the chromatic scale, they are not suited to reflect quintal relationships and therefore are not adequate for illustrating the concepts introduced by the Harmonic Processions theory. For example, the A and B suffixes do not align with sharp- and flat-projecting sets (Chapter 16), and the Forte sequence itself, unsurprisingly, does not reflect the pathway of the Natural Harmonic Procession.

Nevertheless, because Allen Forte's numbering system (commonly referred to as the Forte numbers) has become standard among music theorists, we cite these numbers alongside the Harmonic-Processions numbers in order to maintain clarity and continuity.

[31] Allen Forte, *The Structure of Atonal Music* (New Haven: Yale University Press, 1973).

[32] Forte's prime form here is referred to as chromatic prime form to distinguish it from the quintal prime form in the Harmonic Processions.

[33] HP number 49#

Tables 47 and 48 list all sets sorted according to the Forte sequence and serve as a convenient reference when working with the Forte numbering system.

Reverse Lookup of Set Numbers

If we wish to work with a particular sonority but cannot locate it in the Harmonic Processions tables and its Forte number is unknown, it is helpful to use online pitch-class set calculators (PC-set calculators). When working with such a calculator, proceed as follows:

1. Enter the notes into the calculator; it will return the corresponding Forte number.

2. Consult **Table 47** - Sets Sorted by Forte Number, to determine the set's Natural Harmonic Procession number, its modality, and its other characteristics.

In Practice

- In the enclosed **Tables 47 and 48**, locate the following sets based on their Forte numbers. Note their names and their corresponding Harmonic-Processions numbers.
  - 7-35
  - 8-28
  - 7-31A
  - 6-Z44A
  - 5-24B
  - 4-Z29A

40. Further Thoughts

Symmetry in Dissonance Levels in the Chromatic Scale

The post-tonal theory of music is based largely on the chromatic circle, whereas the Harmonic Processions theory is based on the circle of fifths. These two circular models represent different organizations of notes (pitch classes): the former seeks to equalize consonance and dissonance by treating all interval classes as structurally equivalent, while the latter affirms both tonality and atonality, along with the inherent tension between them as measured in quintal relationships and the interval-class dissonance curve.

In our exposition, the chromatic scale has been given only cursory acknowledgment as the collection of all twelve notes. However, we must highlight the elegance of its symmetrical design, particularly in the distribution of dissonance levels among the interval classes created relative to the starting note. In the example below, we illustrate this phenomenon relative to C, with the dissonance levels corresponding to the hyperbolic dissonance curve discussed in Chapter 31 (Figure 40-1).

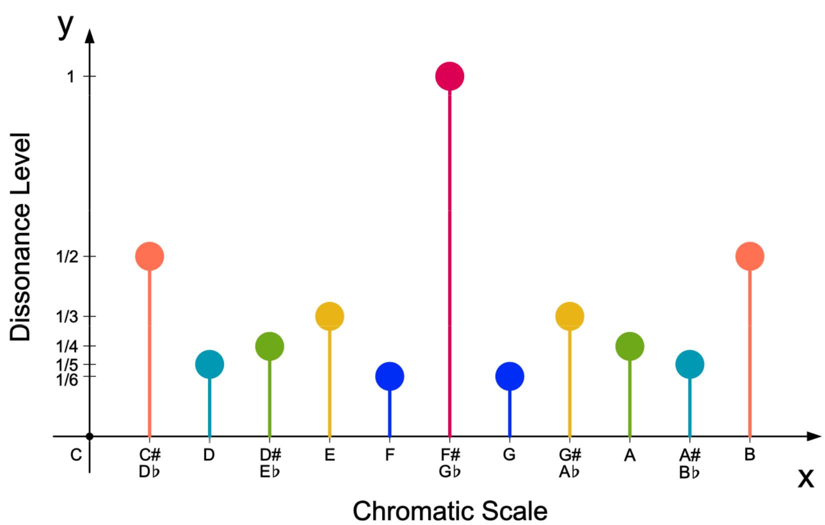


Figure 40-1. Symmetry in dissonance levels in the chromatic scale, relative to C, with hyperbolic curve dissonance values

Implications of the Dissonance Curve for the Circle of Fifths

In Chapter 31 we introduced the concepts of the interval-class gradient and the hyperbolic dissonance curve. These ideas compel us to reexamine the traditional model of the circle of fifths. In its familiar, linear, two-dimensional conception, we envision the circle of fifths exactly as its name suggests: a circle with equally spaced notes around its perimeter (Figure 40-2). But because interval classes, relative to a single reference note, are not arranged on a linear scale but on a hyperbolic curve, the traditional model requires reframing.

If we limit the model to two-dimensional space, we might stretch the circle into an ellipse, placing the more dissonant interval classes farther from the reference note; however, such distortion would misrepresent the adjacent quintal relationships, which remain proportionally constant around the perimeter.

If we instead place the circle of fifths in three-dimensional space (Figure 40-3), we can express the dissonance curve along a third axis (height), such that all notes—relative to a chosen reference note—rise in proportion to their dissonance level relative to that note.

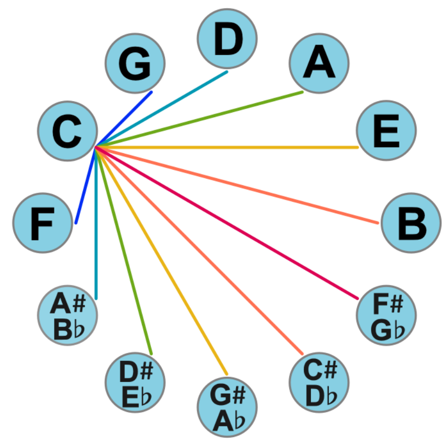


Figure 40-2. Dissonance levels on the circle of fifths, relative to C,
in two-dimensional space

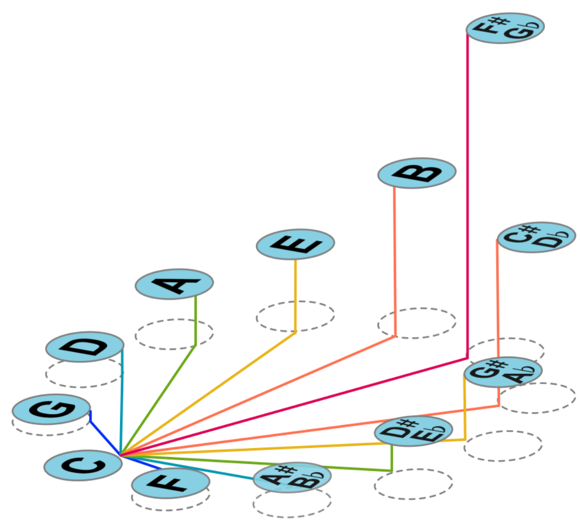


Figure 40-3. Dissonance levels on the circle of fifths, relative to C,
in three-dimensional space, with hyperbolic curve expressed through height

This three-dimensional model is viable as long as we consider one reference note at a time, unless we wish to venture into the more abstract territory in which the circle of fifths exists in a three-dimensional space simultaneously warped in different proportions relative to each of the twelve notes.

This idea echoes certain phenomena in physics, where the same underlying structure yields different geometric descriptions depending on one's vantage point. In general relativity, for example, an observer in free fall can locally describe spacetime as flat, while the same spacetime is globally curved—the intrinsic geometry is fixed, yet its experienced character shifts with one's frame.

In gauge theory, a field may be expressed in many mathematically distinct but physically equivalent forms depending on the gauge chosen; the observable content remains invariant, but the description's shape changes.

Likewise, the harmonic space defined by the circle of fifths remains topologically stable, yet its geometry shifts when viewed through the dissonance curve of interval classes in relation to a particular note. Each reference note reshapes the landscape, producing a family of overlapping but distinct "curvatures" that coexist within the same underlying structure.

While it lies beyond the expertise of this author to pursue the full implications of curved space in relation to the circle of fifths, we offer these instinctive reflections in the hope that they may inspire further consideration.

On Harmonic Analysis in Music Notation

The traditional music-notation system, as it has been handed down for centuries, was created with performers in mind, and in that role it has served adequately. One could criticize the challenges posed by multiple clefs, elaborate key signatures, or transposing instruments, but at this point little can be done to improve the widely adopted conventions of the five-line staff. For composers and music theorists, however, traditional notation presents a different kind of challenge. Their fluency in reading and writing is not in question; rather, it is the staff's very design that limits harmonic exploration.

Consider the notation of two enharmonically equivalent chords: the all-trichord hexachord on F# and the same on G♭ (120#, F 6-Z17A) (Figure 40-4). They represent the same sonority, yet appear vastly different on the staff. Once these chords are notated in an orchestral score with transposing instruments, their harmonic analysis becomes an exercise akin to solving a Rubik's cube.

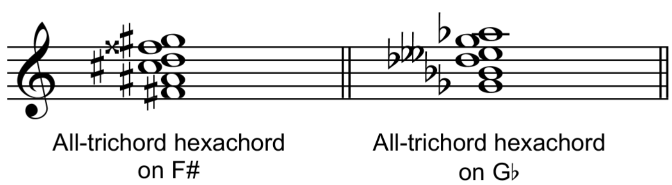


Figure 40-4. All-trichord hexachord on F# and the same on G♭, comparison

This example represents advanced harmony, but even a simple sonority—such as a major triad—can appear in multiple visual variants in music notation (with sharps, flats, double-sharps, double-flats, on the lines, between the lines, etc.). In other words, because the staff offers no single visual representation of a sonority, immediate harmonic analysis is obscured.

There are shorthand systems—Baroque figured bass, the Nashville number system, or the jazz/pop chord-symbol tradition—but all are rooted in the chromatic model, which is antithetical to comparing quintal relationships. While these systems allow us to circumvent the staff, they fall short of conveying immediate harmonic insight when dealing with more complex sonorities.

The construction of the piano keyboard, likewise designed for performers rather than theorists, presents its own obstacles when analyzing harmonic relationships, yet historically has been used to teach music harmony. On the keyboard, the white keys feel accessible and immediate, while the black keys introduce friction—

obstacles to be navigated rather than neutral pitch locations. A C-major chord has a different physical shape from the E♭-major chord. We accept this reality, memorize hand positions, and patiently practice arpeggios, yet transposing a passage or analyzing it harmonically requires significant intellectual effort, even for seasoned musicians.

Traditional notation, traditional and modern number systems, and the design of the keyboard—and indeed all musical instruments—obscure harmonic overview and constrain the free exploration of harmonic possibilities. In addition to this general lack of clarity, the notation system imposes a linear paradigm onto multilayered harmonic structures. When a notation system is primarily melodic and linear in nature, as the five-line staff is, harmonic thinking inevitably becomes dependent on linear inertia. There is nothing inherently wrong with relying on voice leading, but the world of harmony is too rich and too complex to be confined to the convenient, proximity-driven patterns encouraged by traditional notation. We do not propose abolishing the tried-and-true systems that have served performers, conductors, theorists, and composers for centuries. But we do wish to draw attention to their limitations.

Contemporary composers and theorists have the advantage of computers, particularly MIDI editors and notation software, yet these too are built on linear, chromatic assumptions. MIDI editors are somewhat more egalitarian in treatment of individual notes than the staff notation—especially when the superimposed piano grid is removed—but the underlying chromatic model remains the same.

What is needed is a set of tools that allow for rapid harmonic analysis, in which numeric quintal prime forms (or at least scale degrees) can be color-coded relative to the reference note—a kind of movable "Do." With such color application to the note lines on the MIDI grid or to the noteheads in traditional notation software, a major chord would appear consistently the same regardless of its tonality, and any added tones—any change in modality—would be immediately visible through color changes. Imagine the numeric quintal prime form #1,2,5,6,9,10 (the "Ode-to-Napoleon" chord) be immediately distinguishable from #1,2,5,7,8,10 (the Istrian scale) through such color system.

To our knowledge, no such tools currently exist, aside from color-coding the twelve chromatic notes with a fixed "Do" (Cubase by Steinberg, Reaper by Cockos, and to some degree Dorico by Steinberg and Sibelius by Avid Technology), which is of limited use in harmonic analysis. In an era of advanced computing, it is surprising that such tools have not yet emerged; yet we recognize that the market responds to demand, and the demand comes largely from musicians who are unconcerned with the analysis of advanced harmony.

And so we return to the analogy introduced in the opening chapters and confess a certain envy of architects and the wealth of technologies at their disposal—tools that allow them to envision complex three-dimensional structures, whether orthogonal or intricately curved, while simultaneously integrating electrical, plumbing, insulation, and HVAC systems, not to mention modeling the movement of the sun and the shadows cast by a building at any hour of the day and any day of the year.

We wish to end on an optimistic note, expressing the hope that the Harmonic Processions theory will inspire visual thinkers and bring new creative possibilities to composers, improvisers, music theorists, and software engineers.

About the Author

Dosia McKay is an American composer whose music paints deeply immersive and varied atmospheric landscapes. She effortlessly traverses between contemplative serenity, luminous melancholy, and the radiant euphoria of a fully saturated harmonic palette, creating music that feels at once viscerally tangible and transcendent. Drawing on her parallel practice as a visual artist, McKay shapes sound with a painter's command of color and inquisitive openness, crafting musical worlds that dazzle, enchant, and uplift.

McKay's orchestral music has been performed at major American venues including the David H. Koch Theater at Lincoln Center and The John F. Kennedy Center for the Performing Arts. Her works *Unveiling* and *Is Now Not Enough?* were featured by the New York City Ballet in choreography by Sidra Bell with costume design by Christopher John Rogers. Her orchestral premieres include *Watercolors* by the Knoxville Symphony Orchestra at the Tennessee Theatre, *Earthrise* by the Natchitoches–Northwestern Symphony, *Unveiling* by the North/South Consonance Orchestra of New York and the Knox-Galesburg Symphony, and *Farewell Dream Garden* for soprano, flute, and orchestra by the Polish Orpheus Orchestra. Recognized for her emotionally charged and vivid orchestral writing, McKay was commissioned by the Knoxville Symphony to compose *The Lure of the Flowering Fern* for chamber orchestra.

Beyond orchestral writing, her catalog extends to chamber ensembles, choir, solo instruments, and electro-acoustic media. Her music has been featured on National Public Radio and in concerts at the National Gallery of Art in Washington, D.C.; the Festival Gdynia Classica Nova in Poland; the Beijing Modern Music Festival in China; and in performances across Spain, France, Argentina, and the United States, including the Knoxville Museum of Art, the Electro-Music Festival, and the Diana Wortham Theater in Asheville. Notable performers include the avant-garde string quartet NeoQuartet, the chamber repertory company Pan Harmonia, the S.E.M. Ensemble, the Spartanburg Philharmonic String Trio, Asheville Ballet, Baroque lutenist Will Tocaben, Argentine guitar virtuoso Sergio Puccini, and many others.

Recent projects include the experimental album *Mystical Piano*; *Days of Innocence* for piano quintet; *Life Shifts*, commissioned by cellist D. Scot Williams; and electro-acoustic albums including *Groundless*, *Endless Immersion* (composed for an electro-acoustic installation), and *Lacrimosa*. With a grant from the North Carolina Arts Council, McKay was commissioned by Pan Harmonia to mark the organization's 20th season with *Rubble Becomes Art*, a song cycle based on the writings of North Carolina women poets. Her string quartet album *Glossolalia*, recorded by NeoQuartet, has been noted by *I Care If You Listen* magazine for its emotional clarity and refusal to conform to stylistic boundaries.

Born in the Baltic city of Gdańsk, Poland, McKay began her musical life as a flutist and improviser before turning to composition in her thirties. She holds an M.M. in Scoring for Film and Multimedia from New York University and a B.M. in Composition from the University of Tennessee.

A Renaissance woman at heart, Dosia McKay has written poetry, short stories, essays, and a novel entitled The Flow.

Harmonic Processions Tables

Table 1 - The Natural Harmonic Procession of All Sets in Dual Exposition

Mod.[1]	Flat-Projecting Sets								HP								Sharp-Projecting Sets	Mod.
--	Perfect Fourth (P4)						E	B	1	F	C						Perfect Fourth (P4)	--
Sus. Triad ♭	Major Second (M2)					A		B	2	F		G					Major Second (M2)	Sus. Triad #
	Suspended Triad					A	E	B	3	F	C	G					Suspended Triad	
Quartal ♭	Minor Third (m3)				D			B	4	F			D				Minor Third (m3)	Quartal #
					D		E	B	5	F	C		D					
	Quartal Tetrachord, Quintal Tetrachord[2]				D	A	E	B	6	F	C	G	D				Quartal Tetrachord, Quintal Tetrachord[3]	
Pentatonic ♭	Major Third (M3)			G				B	7	F				A			Major Third (M3)	Pentatonic #
	Minor Triad			G			E	B	8	F	C			A			Major Triad	
	Whole-Tone Trichord			G		A		B	9	F		G		A			Whole-Tone Trichord	
				G		A	E	B	10	F	C	G		A			Mu Major (c)	
	Major 6th (c)[4], Minor 7th (c)			G	D		E	B	11	F	C		D	A			Major 6th (c), Minor 7th (c)	
	Pentatonic (s), Yo (s), So What (c)			G	D	A	E	B	12	F	C	G	D	A			Pentatonic (s), Yo (s), So What (c)	
Ionian Hexachord ♭	Minor Second (m2)		C					B	13	F					E		Minor Second (m2)	Ionian Hexachord #
			C				E	B	14	F	C				E			
			C			A		B	15	F		G			E			
			C			A	E	B	16	F	C	G			E			
	Phrygian Tetrachord, Locrian Tetrachord[5]		C		D		E	B	17	F	C		D		E		Ionian Tetrachord, Major Tetrachord	
	Aeolian Tetrachord, Dorian Tetrachord[6]		C		D	A		B	18	F		G	D		E		Aeolian Tetrachord, Dorian Tetrachord[7]	
	Aeolian Pentachord		C		D	A	E	B	19	F	C	G	D		E		Ionian Pentachord	
	Major 7th (c), Minor ♭6th (c)		C	G			E	B	20	F	C			A	E		Major 7th (c), Minor ♭6th (c)	
	Minor 9th (c)		C	G		A	E	B	21	F	C	G		A	E		Major 9th (c)	
	Ionian Hexachord, Minor 11th (c)		C	G	D	A	E	B	22	F	C	G	D	A	E		Ionian Hexachord, Minor 11th (c)	
Diatonic ♭	Tritone (TT)	F						B	23	F						B	Tritone (TT)	Diatonic #
		F					E	B	24	F	C					B	Viennese Trichord	
		F				A		B	25	F		G				B	Italian 6th (c)	
		F				A	E	B	26	F	C	G				B		
	Diminished Triad	F			D			B	27	F			D			B	Diminished Triad	
		F			D		E	B	28	F	C		D			B		
	Tristan (c), Half-Diminished 7th (c)[8]	F			D	A		B	29	F		G	D			B	German 6th (c), Dominant 7th (c)	
	Insen (s), Minor 6th 9th (c)	F			D	A	E	B	30	F	C	G	D			B		
	All-Interval Tetrachord	F		G			E	B	31	F	C			A		B	All-Interval Tetrachord	
	Whole-Tone Tetrachord[9]	F		G		A		B	32	F		G		A		B	Whole-Tone Tetrachord[10]	
	Phrygian Pentachord	F		G		A	E	B	33	F	C	G		A		B	Lydian Pentachord	
	Dominant 7th 6th (c)	F		G	D		E	B	34	F	C		D	A		B	Half-Diminished Minor 9th (c)	
	Dominant 9th (c)	F		G	D	A		B	35	F		G	D	A		B	Dominant 9th (c)	
	Dorian Hexachord	F		G	D	A	E	B	36	F	C	G	D	A		B	Lydian Hexachord, Dominant 11th (c)	

Table 15 - The Natural Harmonic Procession of Octachords

HP	Forte	Diss %	Name	Modality	Quintal Prime Form														
76#S	8-23	38.25	Bebop Dominant (s)	Lydian					F	C	G	D	A	E	B	F#			
76♭S	8-23	38.25	Bebop Dominant (s)	Mixolydian				B♭	F	C	G	D	A	E	B				
117#	8-22A	38.96	Bebop Dorian (s), Bebop Minor (s)	Enigmatic					F	C	G	D	A	E	B		C#		
117♭	8-22B	38.96	Eight-Tone Spanish (s)	Mystic			E♭		F	C	G	D	A	E	B				
128#	8-14A	40.24		Blues #					F	C	G	D	A	E		F#	C#		
128♭	8-14B	40.24		Blues ♭			E♭	B♭		C	G	D	A	E	B				
132#	8-16B	43.45		Blues #					F	C	G	D	A		B	F#	C#		
132♭	8-16A	43.45		Blues ♭			E♭	B♭	F		G	D	A	E	B				
133#S	8-6	44.16		Blues #					F	C	G	D		E	B	F#	C#		
133♭S	8-6	44.16		Blues ♭			E♭	B♭	F	C		D	A	E	B				
148#S	8-26	39.17	Bebop Major (s)[11]	Diminished #					F	C	G	D	A	E	B			G#	
148♭S	8-26	39.17	Bebop Major (s)[12]	Diminished ♭		A♭			F	C	G	D	A	E	B				
162#	8-11B	40.38		Hungarian					F	C	G	D	A	E		F#		G#	
162♭	8-11A	40.38		Romanian		A♭		B♭		C	G	D	A	E	B				
170#	8-13A	43.45		Hungarian					F	C	G	D	A		B	F#		G#	
170♭	8-13B	43.45		Romanian		A♭		B♭	F		G	D	A	E	B				
173#	8-Z15B	43.80		Hungarian					F	C	G	D		E	B	F#		G#	
173♭	8-Z15A	43.80		Romanian		A♭		B♭	F	C		D	A	E	B				
175#	8-4A	41.67		Hungarian					F	C	G		A	E	B	F#		G#	
175♭	8-4B	41.67		Romanian		A♭		B♭	F	C	G		A	E	B				
176#	8-27A	42.74	Bebop Melodic Minor (s)	Hungarian					F	C		D	A	E	B	F#		G#	
176♭	8-27B	42.74		Romanian		A♭		B♭	F	C	G	D		E	B				
177#S	8-10	40.03		Hungarian					F		G	D	A	E	B	F#		G#	
177♭S	8-10	40.03		Romanian		A♭		B♭	F	C	G	D	A		B				
181#S	8-20	40.81		Augmented #					F	C	G	D	A	E			C#	G#	
181♭S	8-20	40.81		Augmented ♭		A♭	E♭			C	G	D	A	E	B				
184#	8-Z29B	43.80		Augmented #					F	C	G	D	A		B		C#	G#	
184♭	8-Z29A	43.80		Augmented ♭		A♭	E♭		F		G	D	A	E	B				
186#	8-18A	44.02		Augmented #					F	C	G	D		E	B		C#	G#	
186♭	8-18B	44.02		Augmented ♭		A♭	E♭		F	C		D	A	E	B				
187#	8-19B	41.52		Augmented #					F	C	G		A	E	B		C#	G#	
187♭	8-19A	41.52		Augmented ♭		A♭	E♭		F	C	G		A	E	B				
188#S	8-17	41.17		Augmented #					F	C		D	A	E	B		C#	G#	
188♭S	8-17	41.17		Augmented ♭		A♭	E♭		F	C	G	D		E	B				
190#S	8-8	44.73		Chromatic F#C#G#					F	C	G	D	A			F#	C#	G#	
190♭S	8-8	44.73		Chromatic B♭E♭A♭		A♭	E♭	B♭			G	D	A	E	B				
191#	8-5B	44.87		Chromatic F#C#G#					F	C	G	D		E		F#	C#	G#	
191♭	8-5A	44.87		Chromatic B♭E♭A♭		A♭	E♭	B♭		C		D	A	E	B				

Table 27 - The Modal Harmonic Procession of Heptachords

HP	Forte	Diss. %	Name	Modality	Quintal Prime Form														
42#S	7-35	25.64	Diatonic (s), Major 13th (c)[13]	Diatonic #					F	C	G	D	A	E	B				
42♭S	7-35	25.64	Diatonic (s), Major 13th (c)[14]	Diatonic ♭					F	C	G	D	A	E	B				
66#	7-23B	27.07		Lydian					F	C	G	D	A	E		F#			
73#	7-29A	30.48		Lydian					F	C	G	D	A		B	F#			
75#	7-14B	31.55		Lydian					F	C	G	D		E	B	F#			
66♭	7-23A	27.07		Mixolydian				B♭		C	G	D	A	E	B				
73♭	7-29B	30.48		Mixolydian				B♭	F		G	D	A	E	B				
75♭	7-14A	31.55		Mixolydian				B♭	F	C		D	A	E	B				
96#	7-27A	27.64		Enigmatic					F	C	G	D	A	E			C#		
107#	7-24B	30.98	Enigmatic (s)	Enigmatic					F	C	G	D	A		B		C#		
112#	7-Z36A	31.91		Enigmatic					F	C	G	D		E	B		C#		
114#	7-30B	31.55		Enigmatic					F	C	G		A	E	B		C#		
115#	7-11B	29.06		Enigmatic					F	C		D	A	E	B		C#		
116#S	7-34	29.91	Half-Diminished (s), Acoustic (s)[15]	Enigmatic					F		G	D	A	E	B		C#		
96♭	7-27B	27.64		Mystic			E♭			C	G	D	A	E	B				
107♭	7-24A	30.98		Mystic			E♭		F		G	D	A	E	B				
112♭	7-Z36B	31.91		Mystic			E♭		F	C		D	A	E	B				
114♭	7-30A	31.55	Neapolitan Minor (s)[16]	Mystic			E♭		F	C	G		A	E	B				
115♭	7-11A	29.06		Mystic			E♭		F	C	G	D		E	B				
116♭S	7-34	29.91	Half-Diminished (s), Acoustic (s)[17]	Mystic			E♭		F	C	G	D	A		B				
122#	7-20A	32.12	Persian (s)	Blues #					F	C	G	D	A			F#	C#		
125#	7-5B	32.98		Blues #					F	C	G	D		E		F#	C#		
126#	7-Z18A	32.48		Blues #					F	C	G		A	E		F#	C#		
127#S	7-Z17	29.63		Blues #					F	C		D	A	E		F#	C#		
130#	7-7A	36.04		Blues #					F	C	G	D			B	F#	C#		
131#S	7-15	35.47		Blues #					F	C	G		A		B	F#	C#		
122♭	7-20B	32.12		Blues ♭			E♭	B♭			G	D	A	E	B				
125♭	7-5A	32.98		Blues ♭			E♭	B♭		C		D	A	E	B				
126♭	7-Z18B	32.48		Blues ♭			E♭	B♭		C	G		A	E	B				
127♭S	7-Z17	29.63		Blues ♭			E♭	B♭		C	G	D		E	B				
130♭	7-7B	36.04		Blues ♭			E♭	B♭	F			D	A	E	B				
131♭S	7-15	35.47		Blues ♭			E♭	B♭	F		G		A	E	B				
143#	7-25A	30.84		Diminished #					F	C	G	D	A		B			G#	
146#	7-32B	31.41	Harmonic Major (s), 7-Note Blues (s)[18]	Diminished #					F	C	G	D		E	B			G#	
147#S	7-Z37	29.63		Diminished #					F	C	G		A	E	B			G#	
143♭	7-25B	30.84		Diminished ♭		A♭			F		G	D	A	E	B				
146♭	7-32A	31.41	Harmonic Minor (s)[19]	Diminished ♭		A♭			F	C		D	A	E	B				
147♭S	7-Z37	29.63		Diminished ♭		A♭			F	C	G		A	E	B				
152#S	7-Z12	31.91		Hungarian					F	C	G	D	A			F#		G#	
156#	7-9B	32.41		Hungarian					F	C	G	D		E		F#		G#	

Table 48 - Sets Sorted by Forte Numbers, Including the Numeric Quintal and the Chromatic Prime Form

Forte	HP	Diss. %	Name	Modality	Quintal Prime Form	Numeric Quintal Prime Form	Chromatic Prime Form
2-1	13#S	2.14	Minor Second (m2)	Ionian Hexachord #	FE	#1,6	CC#
2-1	13♭S	2.14	Minor Second (m2)	Ionian Hexachord ♭	CB	♭1,6	CD♭
2-2	2#S	0.85	Major Second (M2)	Suspended Triad #	FG	#1,3	CD
2-2	2♭S	0.85	Major Second (M2)	Suspended Triad ♭	AB	♭1,3	CD
2-3	4#S	1.07	Minor Third (m3)	Quartal #	FD	#1,4	CD#
2-3	4♭S	1.07	Minor Third (m3)	Quartal ♭	DB	♭1,4	CE♭
2-4	7#S	1.42	Major Third (M3)	Pentatonic #	FA	#1,5	CE
2-4	7♭S	1.42	Major Third (M3)	Pentatonic ♭	GB	♭1,5	CE
2-5	1#S	0.71	Perfect Fourth (P4)	N/A	FC	#1,2	CF
2-5	1♭S	0.71	Perfect Fourth (P4)	N/A	EB	♭1,2	CF
2-6	23#S	4.27	Tritone (TT)	Diatonic #	FB	#1,7	CF#
2-6	23♭S	4.27	Tritone (TT)	Diatonic ♭	FB	♭1,7	CG♭
3-1	43#S	5.13	Chromatic Trichord	Lydian	FGF#	#1,3,8	CC#D
3-1	43♭S	5.13	Chromatic Trichord	Mixolydian	B♭AB	♭1,3,8	CD♭D
3-2A	15#	4.06		Ionian Hexachord #	FGE	#1,3,6	CC#D#
3-2B	15♭	4.06		Ionian Hexachord ♭	CAB	♭1,3,6	CDE♭
3-3A	45♭	4.63		Mixolydian	B♭DB	♭1,4,8	CD♭E
3-3B	45#	4.63		Lydian	FDF#	#1,4,8	CD#E
3-4A	14♭	4.27		Ionian Hexachord ♭	CEB	♭1,2,6	CD♭F
3-4B	14#	4.27		Ionian Hexachord #	FCE	#1,2,6	CEF
3-5A	24#	7.12	Viennese Trichord	Diatonic #	FCB	#1,2,7	CC#F#
3-5B	24♭	7.12		Diatonic ♭	FEB	♭1,2,7	CFG♭
3-6	9#S	3.13	Whole-Tone Trichord	Pentatonic #	FGA	#1,3,5	CDE
3-6	9♭S	3.13	Whole-Tone Trichord	Pentatonic ♭	GAB	♭1,3,5	CDE
3-7A	5#	2.64		Quartal #	FCD	#1,2,4	CDF
3-7B	5♭	2.64		Quartal ♭	DEB	♭1,2,4	CE♭F
3-8A	25#	6.55	Italian 6th (c)	Diatonic #	FGB	#1,3,7	CDF#
3-8B	25♭	6.55		Diatonic ♭	FAB	♭1,3,7	CEG♭
3-9	3#S	2.28	Suspended Triad	Suspended Triad #	FCG	#1,2,3	CDG
3-9	3♭S	2.28	Suspended Triad	Suspended Triad ♭	AEB	♭1,2,3	CDG
3-10	27#S	6.41	Diminished Triad	Diatonic #	FDB	#1,4,7	CD#F#
3-10	27♭S	6.41	Diminished Triad	Diatonic ♭	FDB	♭1,4,7	CE♭G♭
3-11A	8♭	3.21	Minor Triad	Pentatonic ♭	GEB	♭1,2,5	CE♭G
3-11B	8#	3.21	Major Triad	Pentatonic #	FCA	#1,2,5	CEG
3-12	77#S	4.27	Augmented Triad	Enigmatic	FAC#	#1,5,9	CEG#
3-12	77♭S	4.27	Augmented Triad	Mystic	E♭GB	♭1,5,9	CEA♭
4-1	57#S	9.19	Chromatic Tetrachord[20]	Lydian	FGEF#	#1,3,6,8	CC#DD#
4-1	57♭S	9.19	Chromatic Tetrachord[21]	Mixolydian	B♭CAB	♭1,3,6,8	CD♭DE♭

Thank You

Thank you for your interest in the Harmonic Processions theory. This free PDF edition presents the core theoretical framework. The complete system—including all 40 chapters and the full set of 48 reference tables essential for study and practical use—is available exclusively in the printed edition, which may be purchased at www.DosiaMcKay.com.